Fidelity susceptibility, quantum phase transitions, and quantum adiabatic condition

Speaker: Shi-Jian Gu

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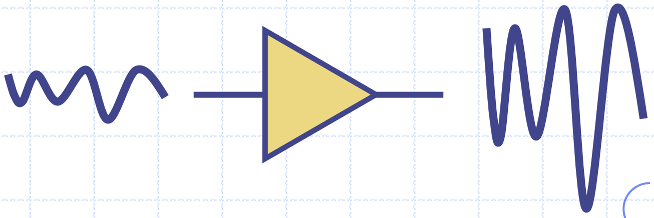
Hong Kong, China



北京大学物理学院，2009年4月9日

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Shu Chen(IoP)
Chang-Pu Sun (ITP)
Xiaoguang Wang(ZJU)
Yuguang Chen(Tongji U.)
Chang-Qing Wu (Fudan U.)

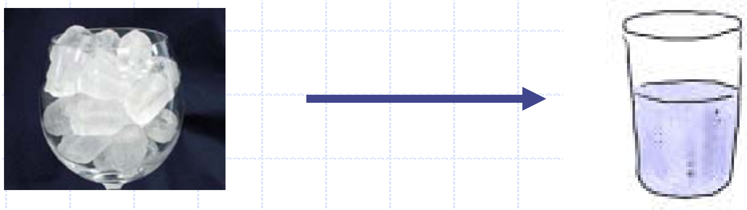
Wen-Qiang Ning (Fudan U.)
Shuo Yang (ITP)
Jian Ma (ZJU)

Introductory review article:
Fidelity approach to quantum phase transitions
Shi-Jian Gu, arXiv:0811.3127

Content

Introduction: QPT

Thermal phase transitions: which is described by non-analytic behaviors of the thermal properties at the transition points, driven by thermal fluctuation.

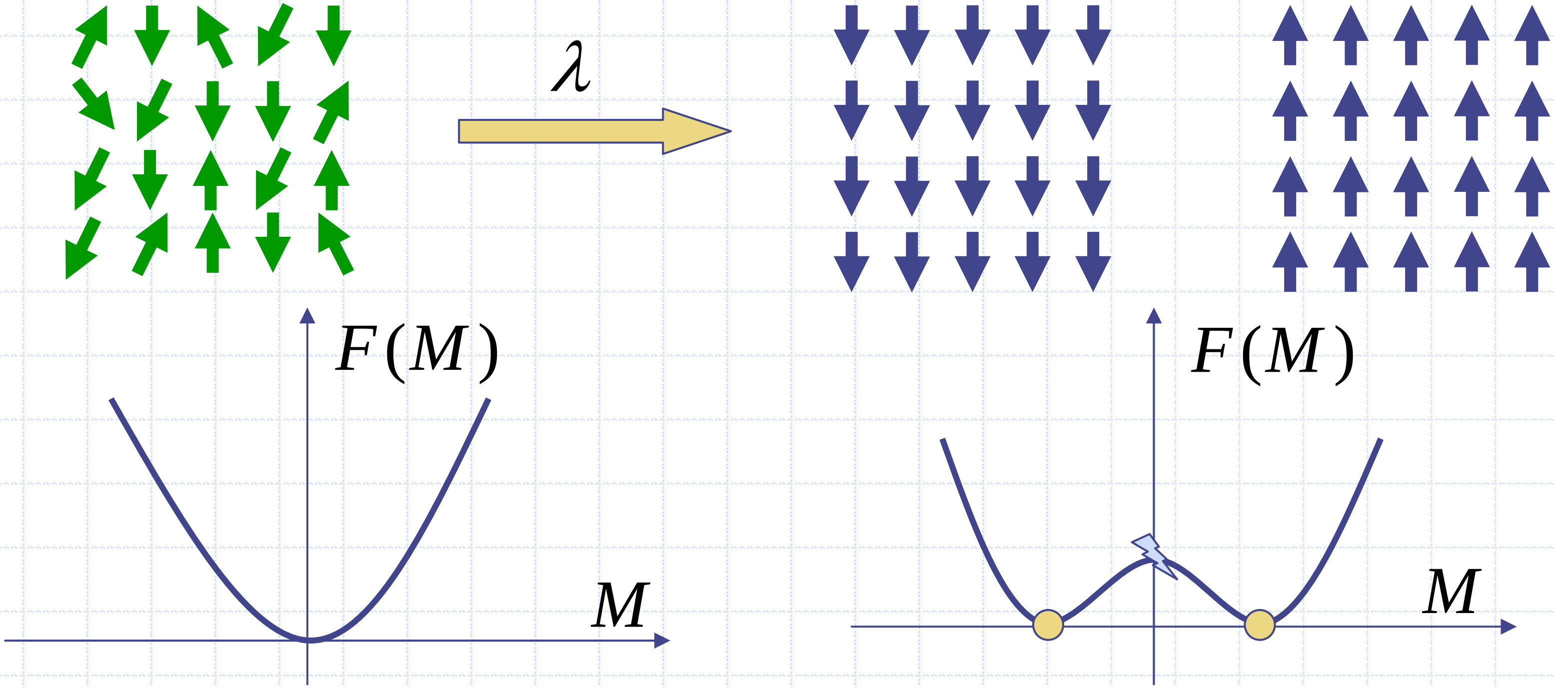


Quantum phase transitions: driven by the quantum fluctuations and are described by the non-analytic behaviors of the ground-state properties at the transition points.

- Mott-insulator transition in Hubbard model.
- Doped High-Tc superconductor

Introduction: traditional method

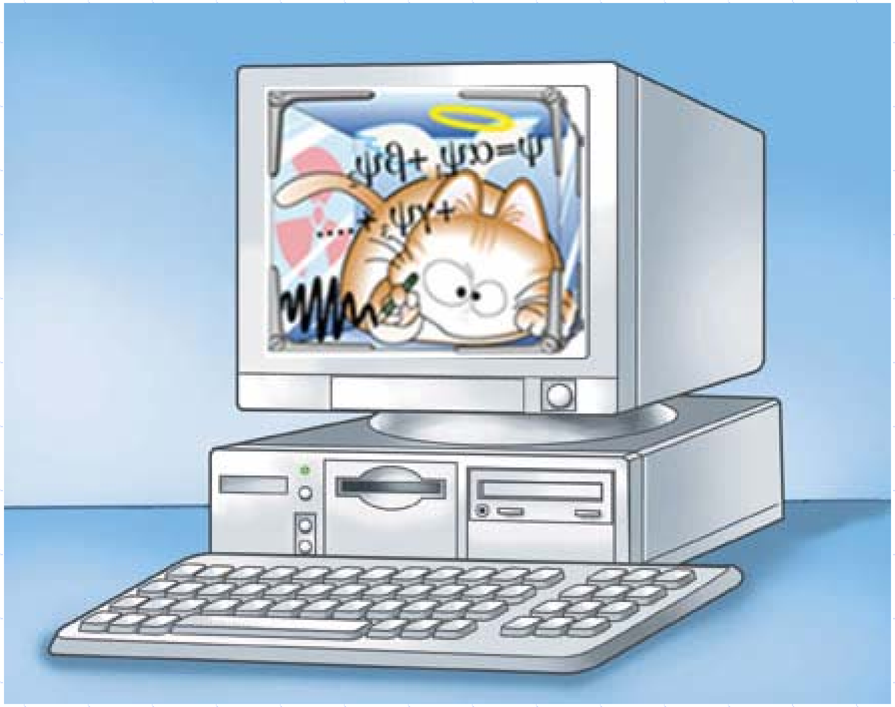
Landau's symmetry-breaking theory



$F(M) = AM^2 + BM^4 +$ $\qquad$ $F(M) = AM^2 + BM^4 +$

$A > 0, B > 0$ $\qquad$ $A < 0, B > 0$

Introduction: quantum information

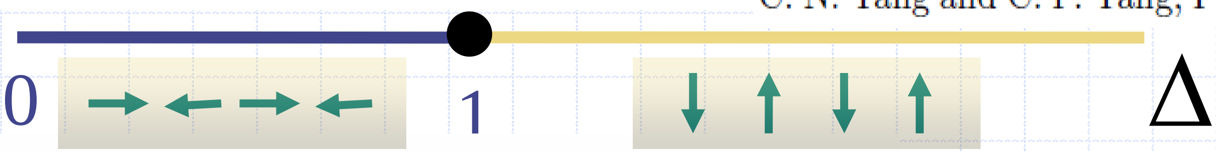


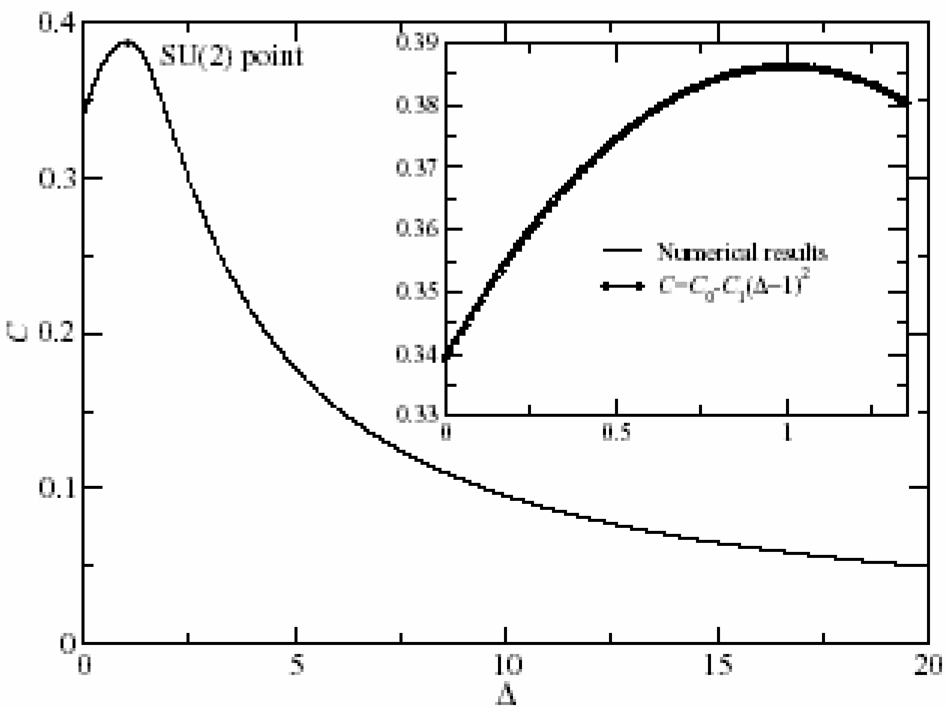
A practic quantum computer seems still a dream, but the development in quantum information science has shed new lights on other related fields.

Introduction: QPT & quantum entanglement

$$\hat{H}_{\mathrm{XXZ}} = \sum_i \left(\hat{s}_i^x \hat{s}_{i+1}^x + \hat{s}_i^y \hat{s}_{i+1}^y + \Delta \hat{s}_i^z \hat{s}_{i+1}^z \right);$$

C. N. Yang and C. P. Yang, Phys. Rev. **150**, 321 (1966)





$$C_{ij} \approx \left\langle s_i^x s_j^x \right\rangle + \left\langle s_i^x s_j^x \right\rangle + \left\langle s_i^x s_j^x \right\rangle$$

$$C = C_0 - C_1 (\Delta - 1)^2$$

$$C_0 = 2\ln 2 - 1$$

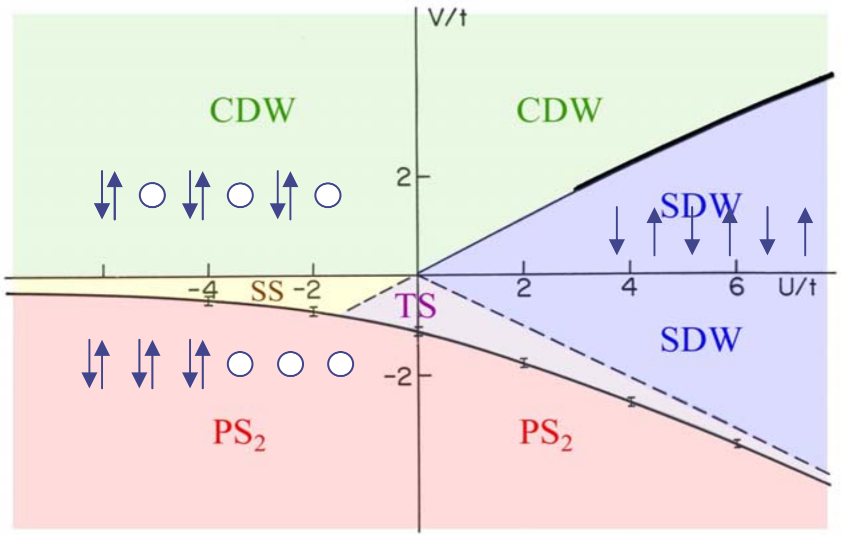
$$C_1 = 2\ln 2 - \frac{1}{2} - \frac{2}{\pi} - \frac{2}{\pi^2}$$

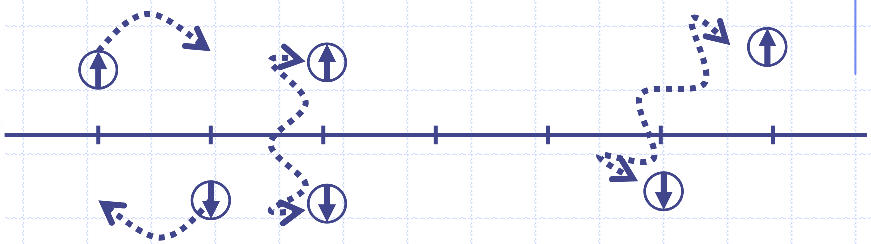
Gu etal, PRA, 68, 042330(2003).

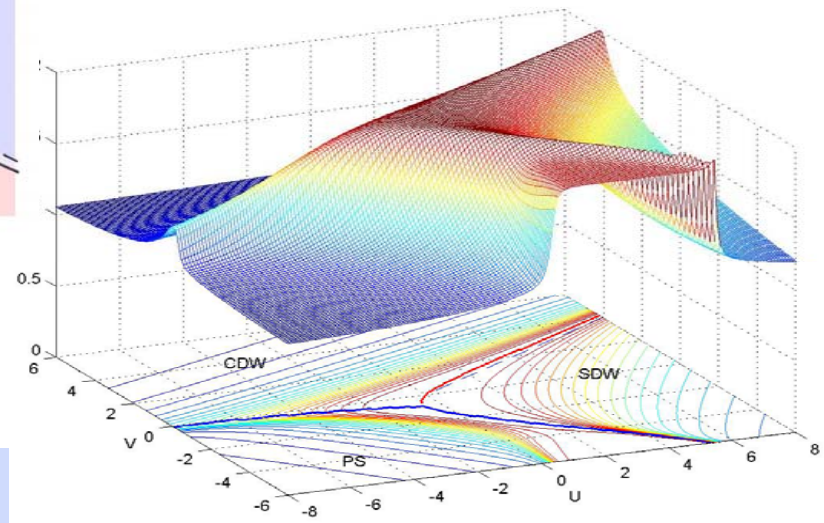
Introduction: QPT & quantum entanglement

The extended Hubbard model

$$H = -\sum_{\sigma,j,\delta} c^{+}_{j,\sigma} c_{j+\delta,\sigma} + U\sum_{j} n_{j,\uparrow} n_{j,\downarrow} + V\sum_{j} n_j n_{j+1}$$







Gu, etal, PRL. 93, 086402 (2004).

Introduction: QPT & quantum entanglement

Detecting Topological Order in a Ground State Wave Function

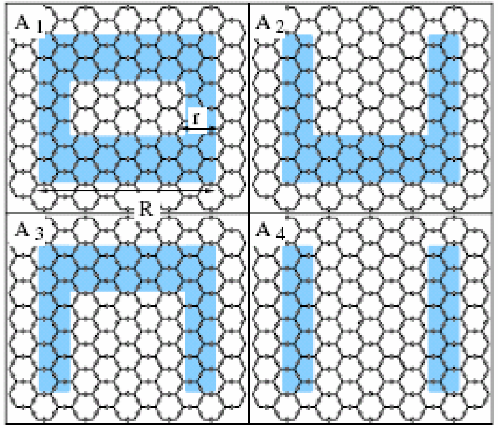
Kitaev&Preskill

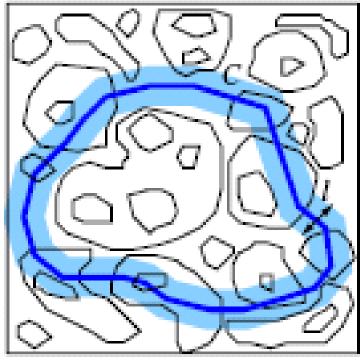
Michael Levin and Xiao-Gang Wen

Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA

(Received 25 October 2005; published 24 March 2006)

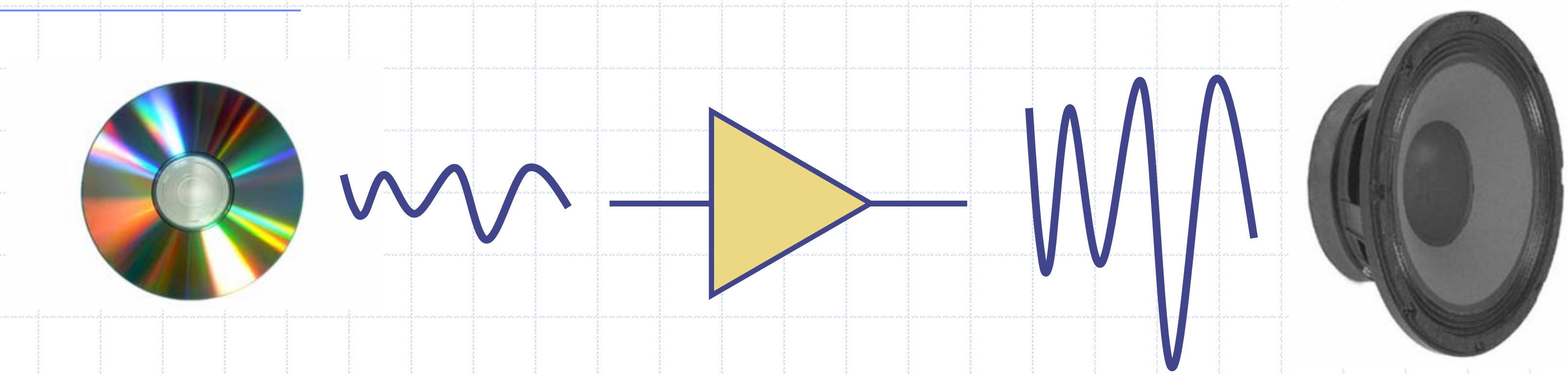
A large class of topological orders can be understood and classified using the string-net condensation picture. These topological orders can be characterized by a set of data $(N, d_i, F^{ijk}_{lmn}, \delta_{ijk})$. We describe a way to detect this kind of topological order using only the ground state wave function. The method involves computing a quantity called the "topological entropy" which directly measures the total quantum dimension $D = \sum_i d_i^2$.





$$S(\rho) = \alpha L - \gamma + \cdots$$

Introduction: classical fidelity



Definition

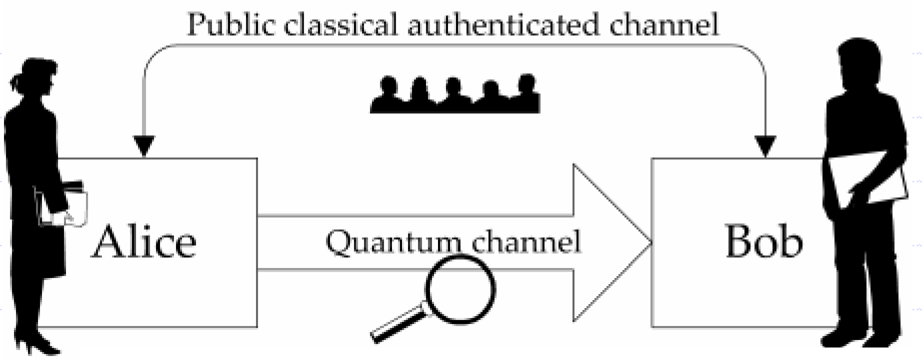
$$\rho = p_1|1\rangle\langle 1| + p_2|2\rangle\langle 2| + \cdots + p_N|N\rangle\langle N|$$

$$\sigma = q_1|1\rangle\langle 1| + q_2|2\rangle\langle 2| + \cdots + q_N|N\rangle\langle N|$$

$$F = \sum_i \sqrt{p_i q_i}$$

Introduction: quantum fidelity

$|\Psi\rangle$



$|\Psi'\rangle$

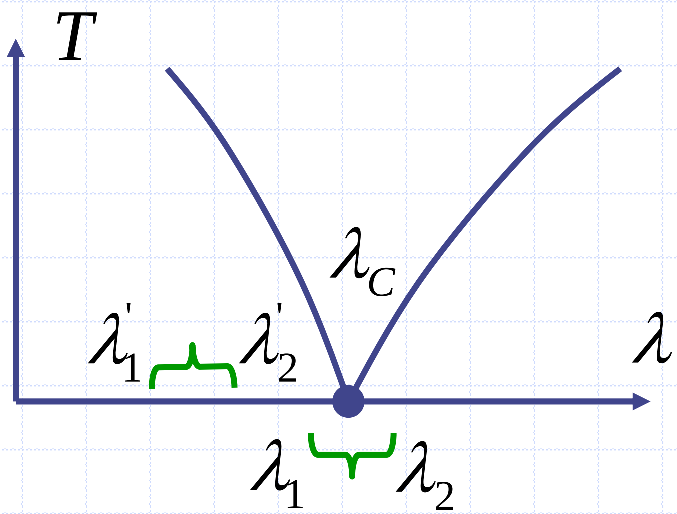
$$F(\Psi', \Psi) = |\langle \Psi' | \Psi \rangle| \qquad \mathbf{a} \cdot \mathbf{b} = ab\cos(\theta)$$

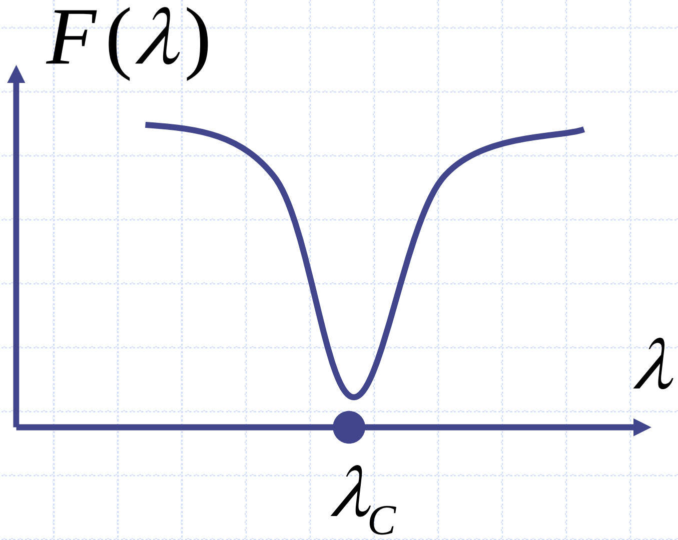
A. Uhlmann, Rep. Math. Phys. 9, 273 (1976)
R. Jozsa, J. Mod. Opt. 41, 2315 (1994).

$$|\Psi(\theta)\rangle = \cos\theta|\uparrow\rangle + \sin\theta|\downarrow\rangle,$$
$$|\Psi(\theta')\rangle = \cos\theta'|\uparrow\rangle + \sin\theta'|\downarrow\rangle,$$

$$F(\Psi(\theta'), \Psi(\theta)) = |\cos(\theta - \theta')|$$

Introduction: information perspective





$$\langle \psi(\lambda_1') | \psi(\lambda_2') \rangle$$

$$< \langle \psi(\lambda_1) | \psi(\lambda_2) \rangle$$

$$\delta\lambda = \lambda_2 - \lambda_1$$

$$\lambda = (\lambda_2 + \lambda_1)/2$$

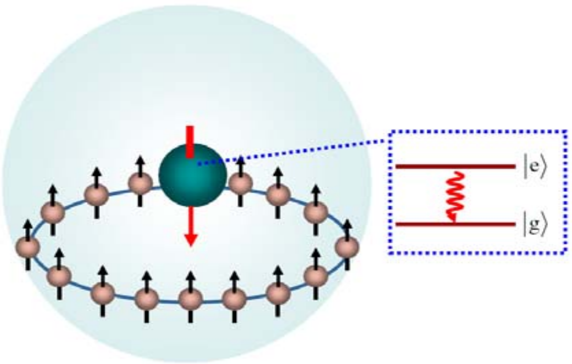
$$F = |\langle \psi(\lambda_1) | \psi(\lambda_2) \rangle|$$

Introduction: QPT & Fidelity

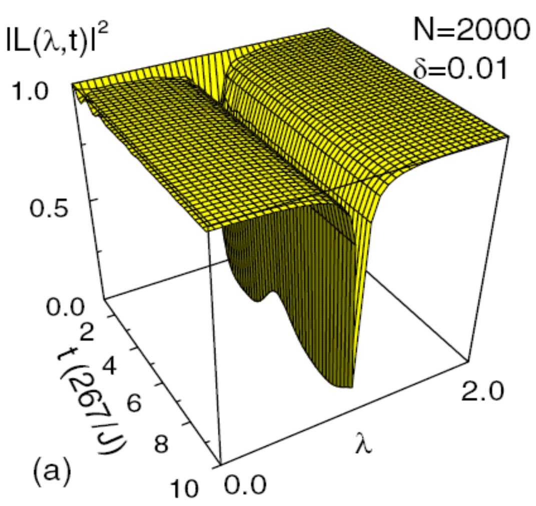
Ising model

H. T. Quan, Z. Song, X. F. Liu, P. Zanardi, and C. P. Sun, Phys. Rev. Lett. **96**, 140604 (2006).

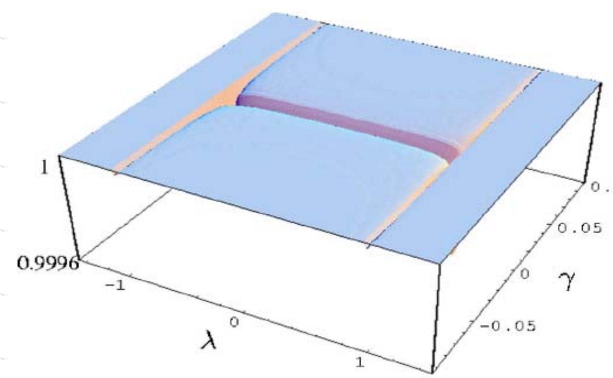
$$H(\lambda, \delta) = -J\sum_j(\sigma_j^z\sigma_{j+1}^z + \lambda\sigma_j^x + \delta|e\rangle\langle e|\sigma_j^x),$$



$$L(\lambda, t) = |\langle\varphi_g(t)|\varphi_e(t)\rangle|^2.$$



$$\hat{H}(\gamma,\lambda) = -\sum_{i=-M}^{M}\left(\frac{1+\gamma}{2}\hat{\sigma}_i^x\hat{\sigma}_{i+1}^x + \frac{1-\gamma}{2}\hat{\sigma}_i^y\hat{\sigma}_{i+1}^y + \frac{\lambda}{2}\hat{\sigma}_i^z\right).$$



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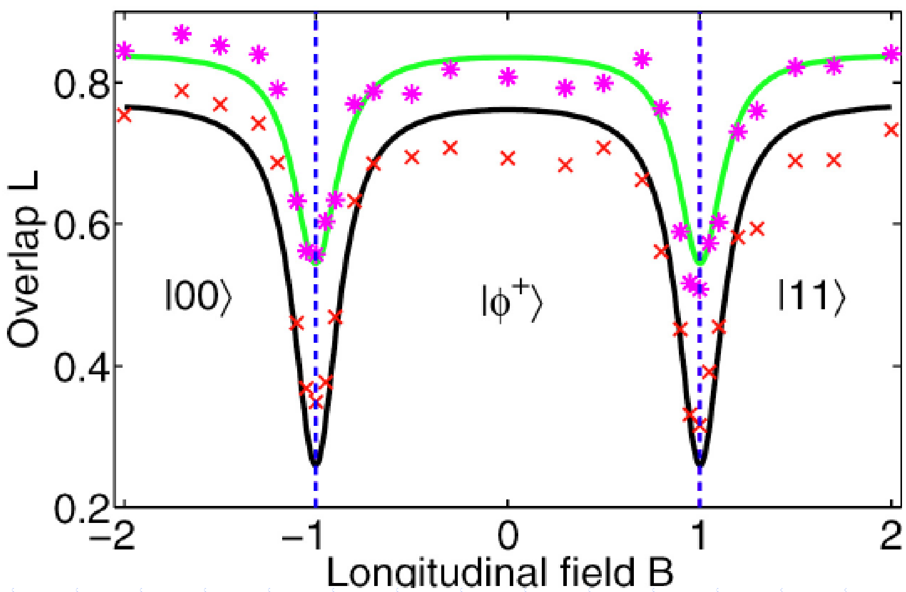
Introduction: QPT & Fidelity

Nuclear-magnetic-resonance(NMR) experiments

J. Zhang, X. Peng, N. Rajendran, and D. Suter, Phys. Rev. Lett. **100**, 100501 (2008).

$$H^s = \sigma_1^z \sigma_2^z + B_x(\sigma_1^x + \sigma_2^x) + B_z(\sigma_1^z + \sigma_2^z)$$



J. Zhang, etal, Phys. Rev. A 79, 012305 (2009)

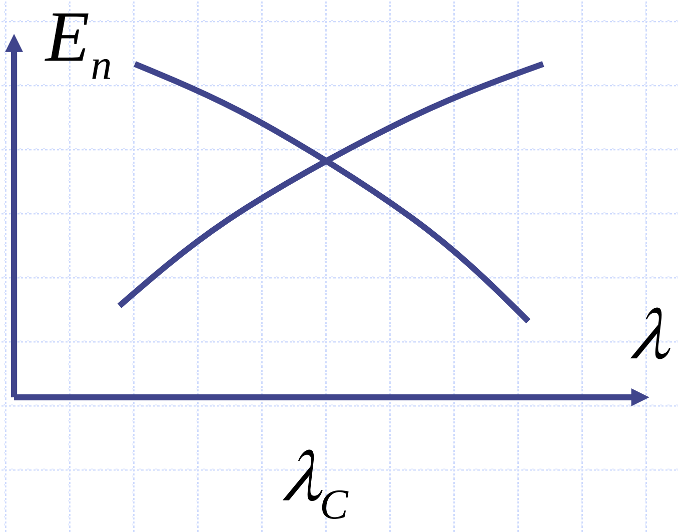
Content

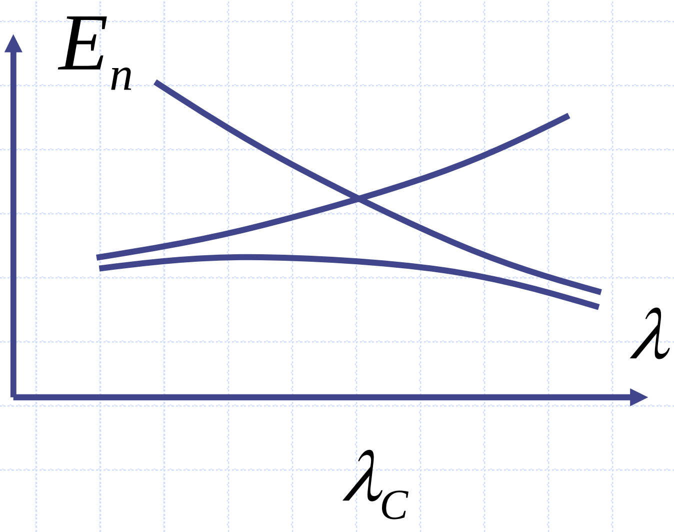
Spectra reconstruction

How does a QPT happen for a general quantum system

$$H(\lambda) = H_0 + \lambda H_I,$$

$$H(\lambda)|\Psi_n(\lambda)\rangle \;=\; E_n|\Psi_n(\lambda)\rangle$$





Perturbation method in quantum mechanics

You, Li, and Gu, PRE, 76, 022101 (2007)

Fidelity susceptibility

$$|\Psi_0(\lambda+\delta\lambda)\rangle = |\Psi_0(\lambda)\rangle + \delta\lambda \sum_{n\neq 0} \frac{H_{n0}(\lambda)|\Psi_n(\lambda)\rangle}{E_0(\lambda) - E_n(\lambda)}$$

$$H_{n0} = \langle\Psi_n(\lambda)|H_I|\Psi_0(\lambda)\rangle.$$

$$F_i(\lambda,\delta) = |\langle\Psi_0(\lambda)|\Psi_0(\lambda+\delta)\rangle|$$

$$F^2 = 1 - \delta\lambda^2 \sum_{n\neq 0} \frac{|\langle\Psi_n(\lambda)|H_I|\Psi_0(\lambda)\rangle|^2}{[E_n(\lambda) - E_0(\lambda)]^2} + \cdots$$

$$\chi_F \equiv \lim_{\delta\lambda\to 0} \frac{-2\ln F_i}{\delta\lambda^2}$$

$$\chi_F(\lambda) = \sum_{n\neq 0} \frac{|\langle\Psi_n(\lambda)|H_I|\Psi_0(\lambda)\rangle|^2}{[E_n(\lambda) - E_0(\lambda)]^2}$$

Perturbation method in quantum mechanics

Fidelity susceptibility: what is the physics

$$\chi_F(\omega) = \sum_{n\neq 0} \frac{|\langle\Psi_n|H_I|\Psi_0\rangle|^2}{[E_n - E_0]^2 + \omega^2}$$

You, Li, and Gu, PRE, 76, 022101 (2007)

$$\chi_F = \int \tau \left[\langle\Psi_0|H_I(\tau)H_I(0)|\Psi_0\rangle - \langle\Psi_0|H_I|\Psi_0\rangle^2\right] d\tau$$

Fidelity susceptibility <==> dynamic structure factor

$$[H_0, H_I] = 0 \longrightarrow \chi_F = 0$$

Extension to thermal phase transitions

Fidelity susceptibility: extension to TPT

P. Zanardi, H. T. Quan, X. Wang, and C. P. Sun, Phys. Rev. A **75**, 032109 (2007).

$$F_i(\beta,\delta) = \frac{Z(\beta)}{\sqrt{Z(\beta-\delta\beta/2)Z(\beta+\delta\beta/2)}},$$

You, Li, and Gu, PRE, 76, 022101 (2007)

$$\chi_F = \left.\frac{-2\ln F_i}{\delta\beta^2}\right|_{\delta\beta\to 0} = \frac{C_v}{4\beta^2} \qquad C_v = \beta^2(\langle E^2\rangle - \langle E\rangle^2)$$

$$\chi_F = \left.\frac{-2\ln F_i}{\delta h^2}\right|_{\delta h\to 0} = \frac{\beta\chi}{4} \qquad \chi = \beta(\langle \dot{M}^2\rangle - \langle \dot{M}\rangle^2)$$

A neat connection between quantum information theoretic concepts and thermodynamic quantities

八卦一下

"fidelity susceptibility" Google 搜索 高级搜索 | 使用偏好

所有网页 中文网页 简体中文网页

网页 约有 **735** 项符合 **"fidelity susceptibility"** 的查询结果，以下是第**1-10** 项 （搜索用时 **0.17** 秒）

小提示： 只搜索**中文(简体)**查询结果，可在 使用偏好 指定搜索语言

Shi-Jian Gu 's Homepage

Quantum criticality of the Lipkin-Meshkov-Glick Model in terms of fidelity susceptibility. Ho-Man Kwok, Wen-Qiang Ning, Shi-Jian Gu, and Hai-Qing Lin, Phys. ...

www.phy.cuhk.edu.hk/people/teach/sjgu/publication.htm - 40k - 网页快照 - 类似网页

Sciencepaper Online - [翻译此页]

We analyze the critical properties of Lipkin-Meshcov-Glick model in terms of fidelity susceptibility through -body reduced density matrix which we regard as ...

www.paper.edu.cn/en/paper.php?serial_number=200901-405 - 21k - 网页快照 - 类似网页

APS - 2009 APS March Meeting - Event - Fidelity susceptibility and ... - [翻译此页]

In this talk, I will introduce the quantum fidelity approach to quantum phase transitions based on its leading term, i.e. the fidelity susceptibility. ...

meetings.aps.org/Meeting/MAR09/Event/97293 - 类似网页

Universality class described by the FS

$$H_I = \sum_r V(r)$$

L. C. Venuti and P. Zanardi, Phys. Rev. Lett. **99**, 095701 (2007).

$$\chi_F = \int \tau \left[\langle\Psi_0|H_I(\tau)H_I(0)|\Psi_0\rangle - \langle\Psi_0|H_I|\Psi_0\rangle^2\right] d\tau$$

$$r' = s\,r, \quad \tau' = s^{\zeta}\tau, \quad V(r') = s^{-\Delta_V} V(r)$$

$$\frac{\chi_F}{L^d} \sim L^{d+2\zeta-2\Delta_V} \qquad \mu = 2d + 2\zeta - 2\Delta_V \qquad \chi_F \approx L^{\mu}$$

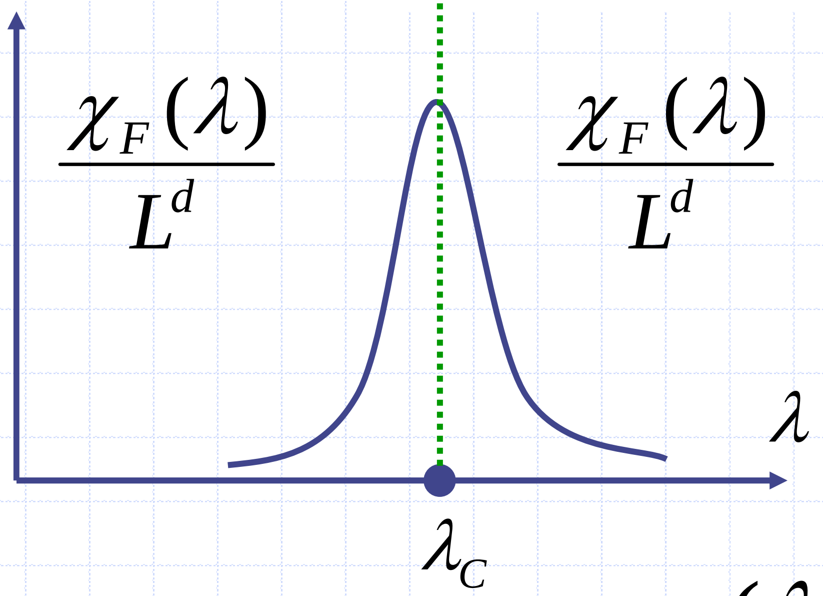
S. J. Gu, H. M. Kwok, W. Q. Ning, and H. Q. Lin, Phys. Rev. B **77**, 245109 (2008). arXiv:0706.2495

$$\frac{\chi_F}{L^d} \sim \frac{1}{|\lambda - \lambda_c|^{\alpha}} \qquad \alpha = \frac{\mu - d}{\nu}$$

Dimension of fidelity susceptibility

The fidelity susceptibility

$$\frac{\chi_F}{L^d} \sim \frac{1}{|\lambda - \lambda_c|^\alpha} \qquad \alpha = \frac{d + 2\zeta - 2\Delta_V}{\nu}$$



$$\chi_F(\lambda) \propto L^d \quad ?$$

ITP, Department of Physics, CUHK

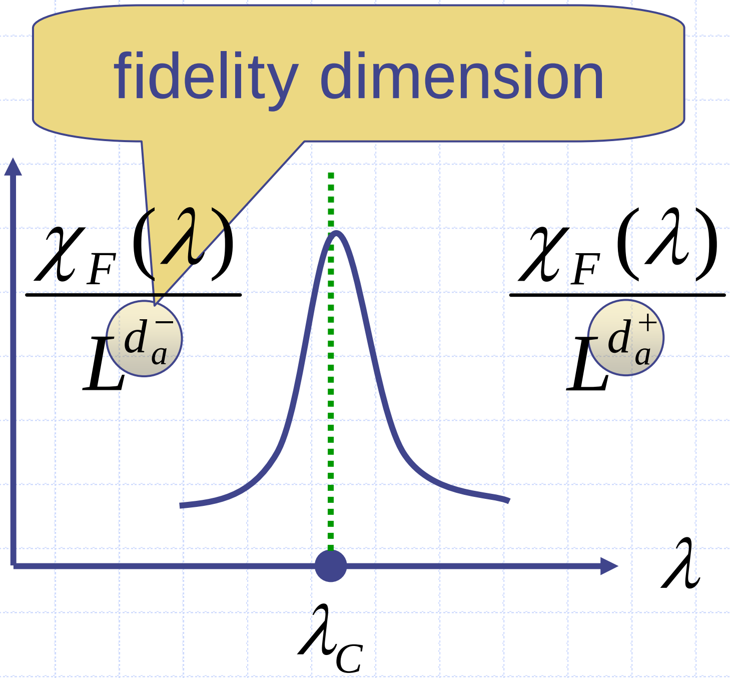
Quantum adiabatic dimension

In general quantum phases

Gu and Lin, arXiv: 08073491

$$C(r,\tau) = \frac{1}{r^{2\Delta_V}} f(r\tau^{1/\zeta})$$

$$\begin{aligned}
\frac{\chi_F}{L^d} &= \sum_r \int \frac{\tau}{r^{2\Delta_V}} f(r\tau^{1/\zeta}) d\tau, \\
&\sim \sum_r \frac{1}{r^{2\Delta_V - 2\zeta}} \\
&\propto \begin{cases} L^{d+2\zeta-2\Delta_V}, & 2\Delta_V - 2\zeta \neq d \\ \ln L, & 2\Delta_V - 2\zeta = d \end{cases}
\end{aligned}$$



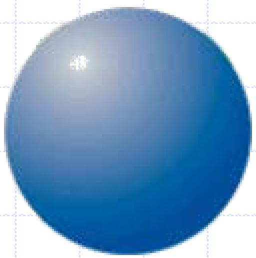
$$\frac{\chi_F}{L^{d_a^\pm}} \sim \frac{1}{|\lambda - \lambda_c|^{\alpha^\pm}} \qquad \frac{\chi(\lambda, L)}{L^{d_a^\pm}} = \frac{A}{L^{-\mu + d_a^\pm} + B(\lambda - \lambda_{\max})^{\alpha^\pm}} \qquad \alpha^\pm = \frac{\mu - d_a^\pm}{\nu}$$

Quantum and classical distance (unpublished)

m

$$f\Delta t = m(v_2 - v_1)$$

$$f\Delta x = \tfrac{1}{2} m(v_2^2 - v_1^2)$$



M

$$F\Delta t = M(v_2 - v_1)$$

$$F\Delta x = \tfrac{1}{2} M(v_2^2 - v_1^2)$$

$$F = ma$$

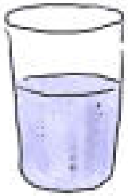
Quantum and classical distance (unpublished)

$$F(\beta_1,\beta_2)=\frac{Z[(\beta_1+\beta_2)/2]}{\sqrt{Z(\beta_1)Z(\beta_2)}}$$

$$\ln F(\beta_1,\beta_2)=\ln Z[(\beta_1+\beta_2)/2]$$
$$-\tfrac{1}{2}\ln Z(\beta_1)-\tfrac{1}{2}\ln Z(\beta_2)$$

$$F=|\langle\psi(\lambda_1)|\psi(\lambda_2)\rangle|$$

$$\frac{\chi_F}{L^d}=\sum_r\int \tau G(r,\tau)d\tau$$



$$\ln F\approx L^{d_a}$$

Logarithmic fidelity

$$\ln F\approx L^d$$

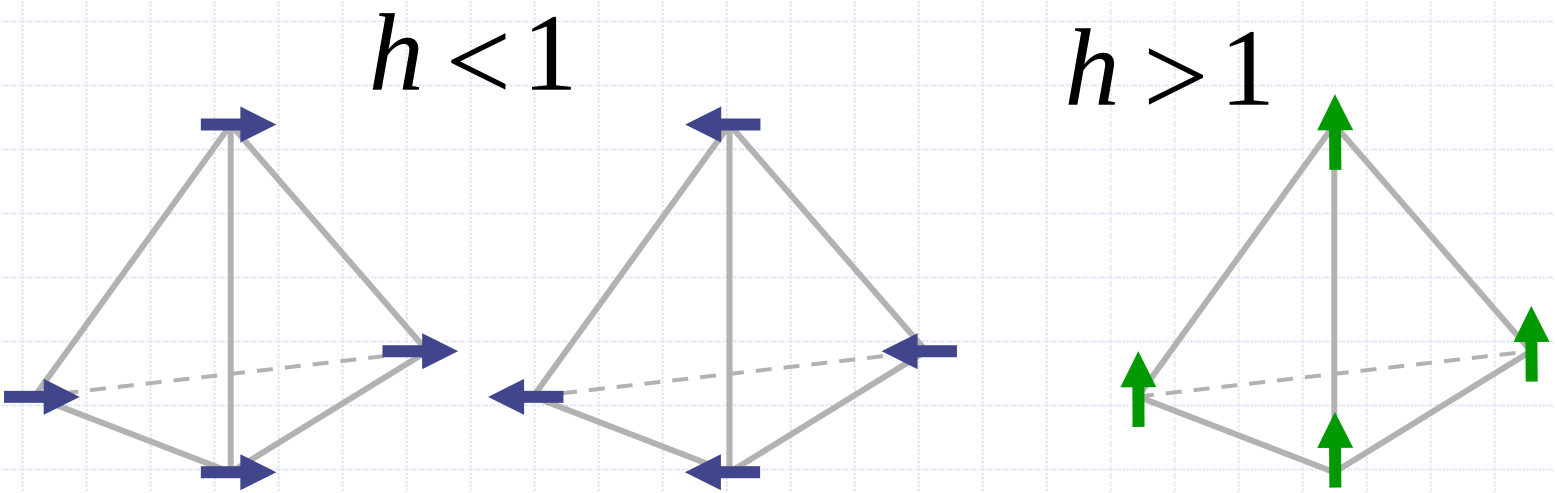
quantum distance can be superextensive

Application: the Lipkin-Meshkov-Glick model

Hamiltonian (N spins)

$$H = -\frac{\lambda}{N}\sum_{i<j}\left(\sigma_x^i\sigma_x^j + \gamma\sigma_y^i\sigma_y^j\right) - h\sum_i \sigma_z^i,$$

Ground phases (ferromagnetic, 4-spin sample)

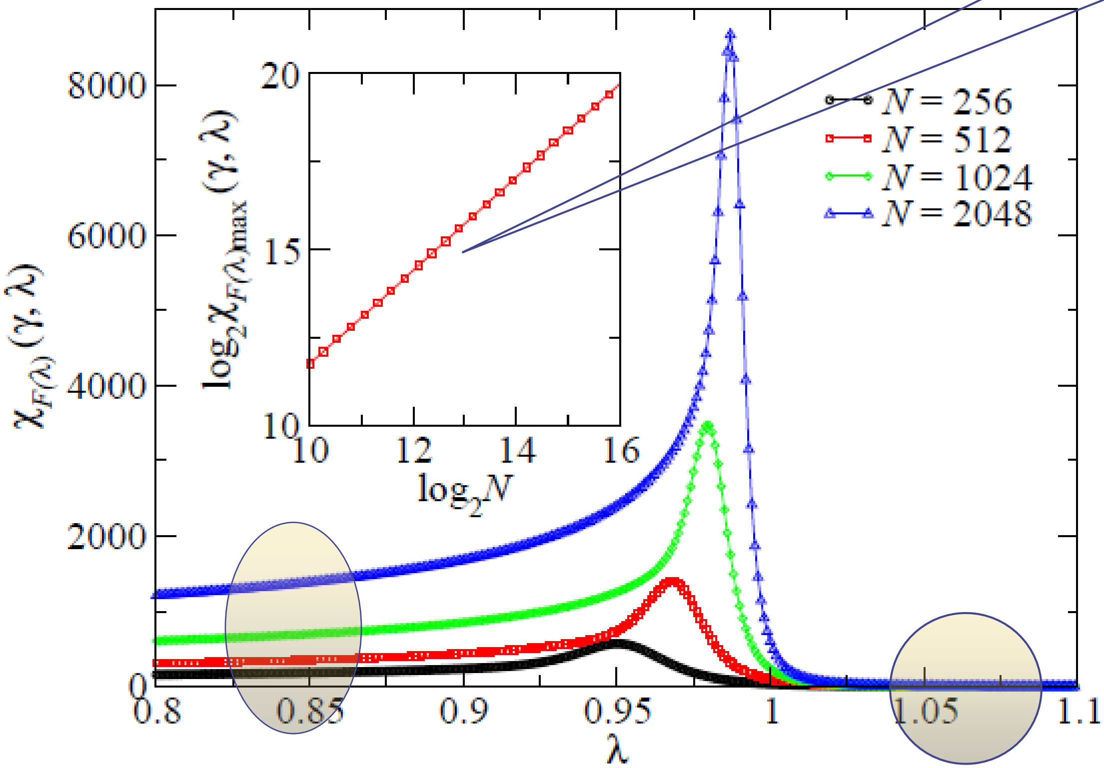


Application: the LMG model

$\gamma = 0.5$

$\chi_{F\max} \propto L^{\mu}$

$\mu \simeq 1.33$



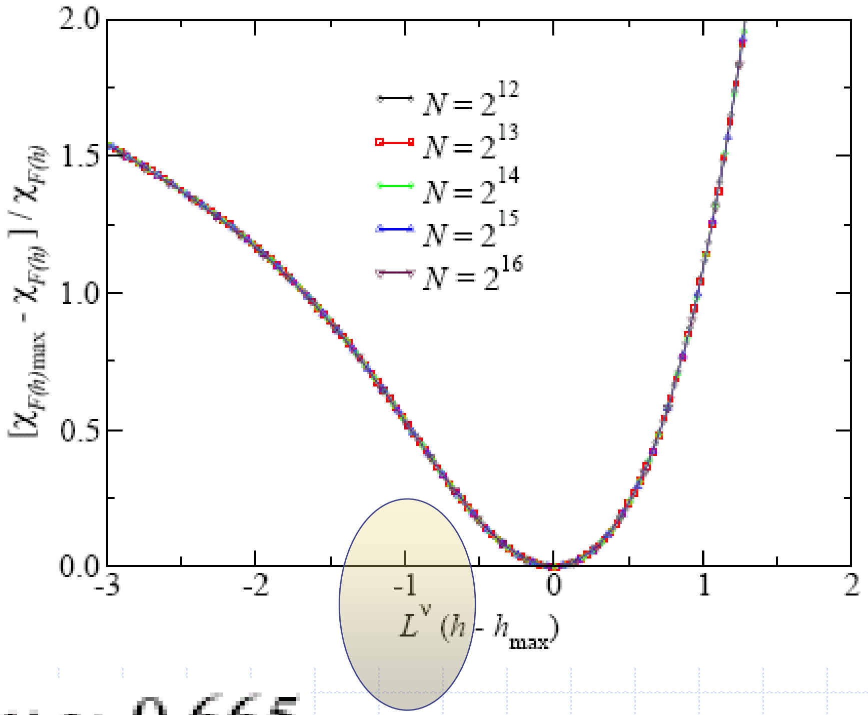
$\chi_F \propto N^0, \quad d_a^+ = 0$

$\chi_F \propto N, \quad d_a^- = 1$

Application: the LMG model

$$\frac{\chi_{F(h)}(h_{\max},\eta)-\chi_{F(h)}(h,\eta)}{\chi_{F(h)}(h,\eta)} = f[L^{\nu}(h-h_{\max})]$$

$$\mu \simeq 1.33 \quad d^{+}=0$$
$$d^{-}=1$$



$$\frac{\chi_F}{L^{d_a^{\pm}}} \sim \frac{1}{|\lambda-\lambda_c|^{\alpha^{\pm}}}$$

$$\alpha^{\pm} = \frac{\mu - d_a^{\pm}}{\nu}$$

$$\alpha^{\pm} = \begin{cases} 1/2 & \lambda < 1 \\ 2 & \lambda > 1 \end{cases}$$

$$\nu \simeq 0.665.$$

Application: the LMG model

J. I. Latorre, R. Orús, E. Rico, and J. Vidal, Phys. Rev. A **71**, 064101 (2005).

If $h > 1$

$$\begin{aligned} S_z &= S - a^\dagger a, \\ S_+ &= (2S - a^\dagger a)^{1/2} a \end{aligned}$$

The Hamiltonian in terms of bosons

$$H = -hN + [2(h-1) + \eta] a^\dagger a - \frac{\eta}{2}\left(a^{\dagger 2} + a^2\right)$$

$$\begin{aligned} a^\dagger &= \cosh(\Theta/2) b^\dagger + \sinh(\Theta/2) b, \\ a &= \sinh(\Theta/2) b^\dagger + \cosh(\Theta/2) b, \end{aligned}$$

The diagonalized form

$$H = -h(N+1) + 2\sqrt{(h-1)(h-1+\eta)}\left(b^\dagger b + \frac{1}{2}\right)$$

Application: the LMG model

H. M. Kwok, W. Q. Ning, S. J. Gu, and H. Q. Lin, Phys. Rev. E **78**, 032103 (2008).

If $h > 1$

$$\chi_{F(h)}(\eta, h > 1) = \frac{\eta^2}{32(h-1)^2(h-1+\eta)^2}$$

If $h < 1$

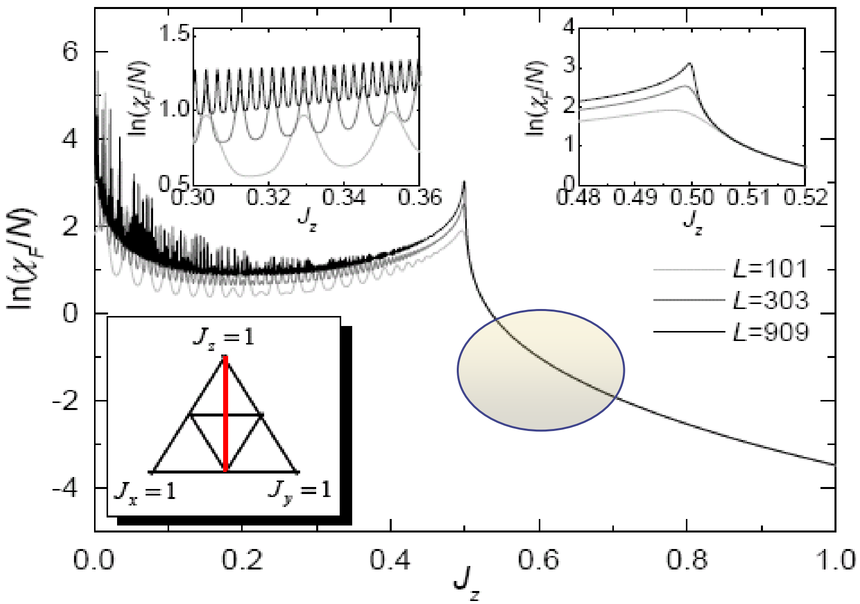
$$\frac{\chi_{F(h)}(\eta, h < 1)}{N} = \frac{1}{4\sqrt{(1-h^2)\eta}}$$

Exponents

$$\alpha = \begin{cases} 2, h > 1 \\ \frac{1}{2}, 0 \leq h < 1 \end{cases}$$

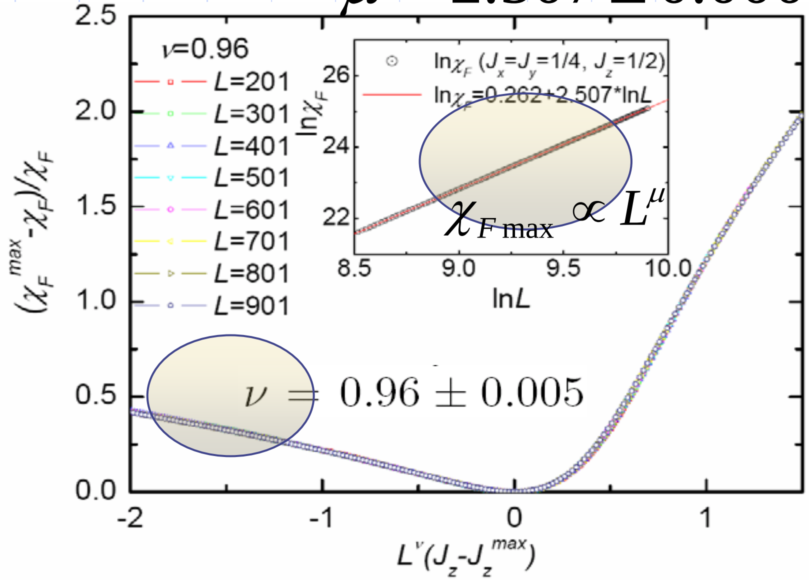
Fidelity susceptibility in topological QPTs

$$\chi_F = \frac{1}{16}\sum_{\mathbf{q}}\left[\frac{\sin q_x + \sin q_y}{\epsilon_{\mathbf{q}}^2 + \Delta_{\mathbf{q}}^2}\right]^2$$



$$\frac{\chi_F}{L^2} \propto \frac{1}{|J_z - J_z^c|^{\alpha}}$$

$$\mu = 2.507 \pm 0.0001$$



$$\frac{\chi_F^{\max} - \chi_F}{\chi_F} = f[L^{\nu}(J_z - J_z^{\max})]$$

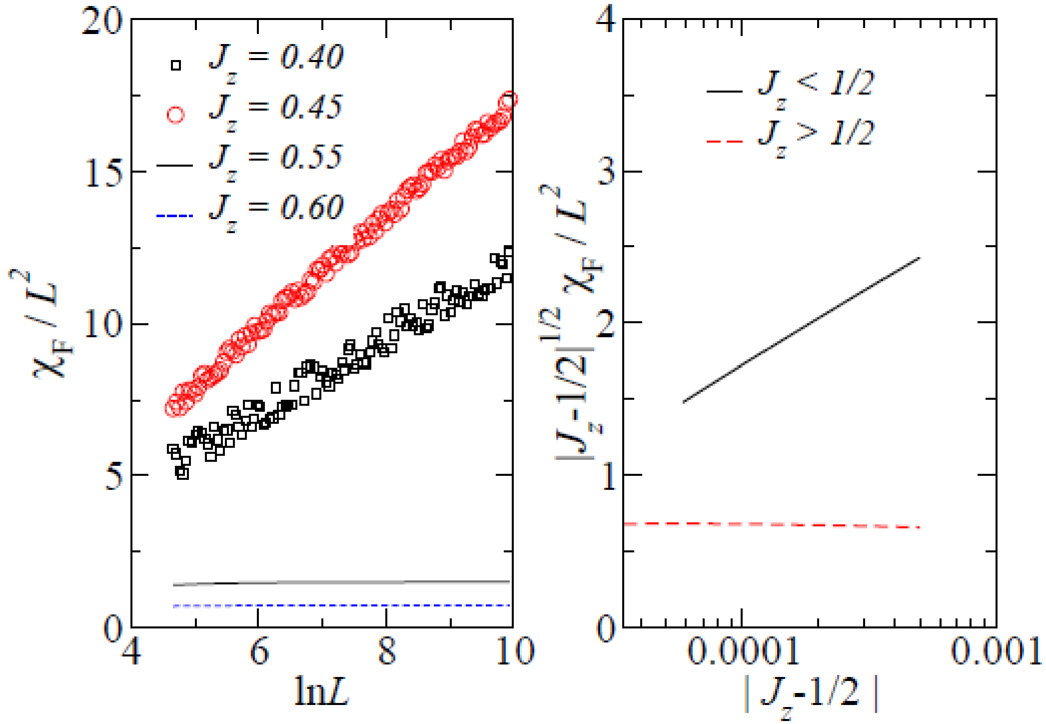
$$\alpha = \frac{\mu - d_a}{\nu} \approx 0.5$$

S. Yang, S. J. Gu, C. P. Sun, and H. Q. Lin, Phys. Rev. A **78**, 012304 (2008).

Fidelity susceptibility in topological QPTs

$$\chi_F = \frac{1}{16}\sum_{\mathbf{q}}\left[\frac{\sin q_x + \sin q_y}{\epsilon_{\mathbf{q}}^2 + \Delta_{\mathbf{q}}^2}\right]^2$$

Gu and Lin, arXiv: 08073491



$J_z > 1/2$

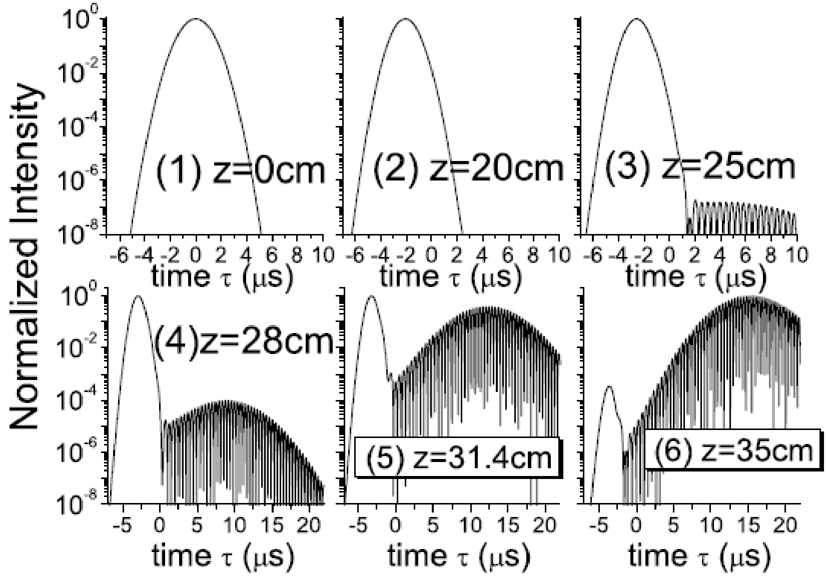
$$\frac{\chi_F}{L^2} \sim \frac{1}{|J_z - J_{z,c}|^{1/2}}$$

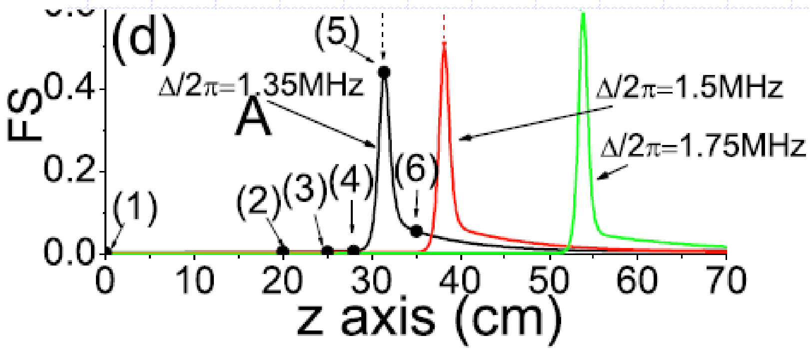
$J_z < 1/2$

$$\frac{\chi_F |J_z - J_{z,c}|^{1/2}}{L^2 \ln L} \sim \ln |J - J_{z,c}|$$

Fidelity susceptibility of pulse in dispersive media

Li-Gang Wang and Shi-Jian Gu, preprint





Content

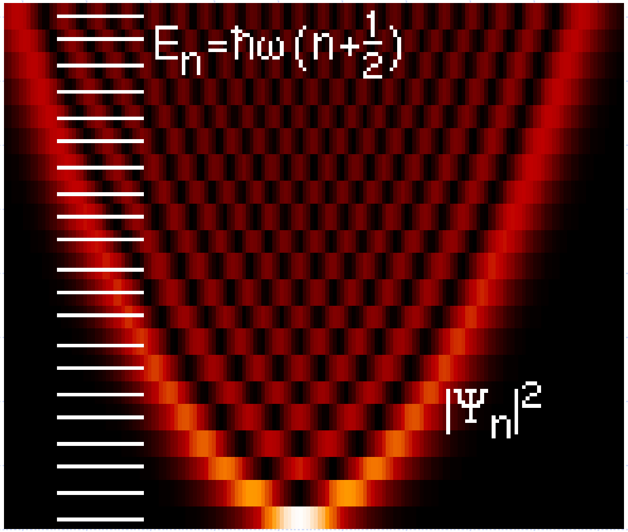
Shi-Jian Gu, arXiv:0902.4623

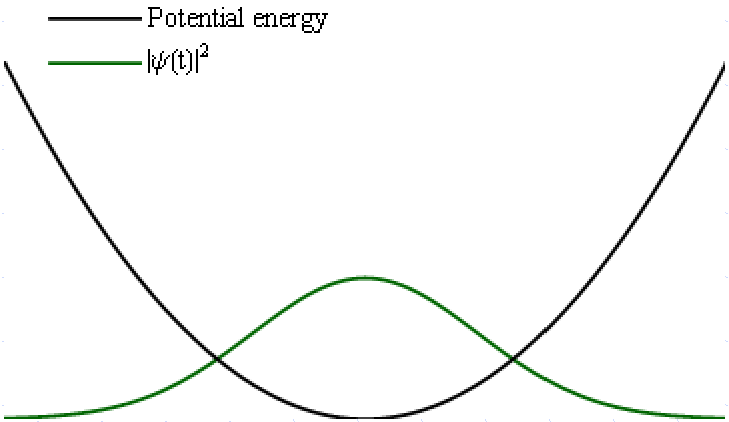
Quantum adiabatic theorem

A physical system remains in its instantaneous eigenstate if a given perturbation is acting on it slowly enough and if there is a gap between the eigenvalue and the rest of the Hamiltonian's spectrum.

$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2$$

$$\omega = \omega_0 t / \tau_0$$

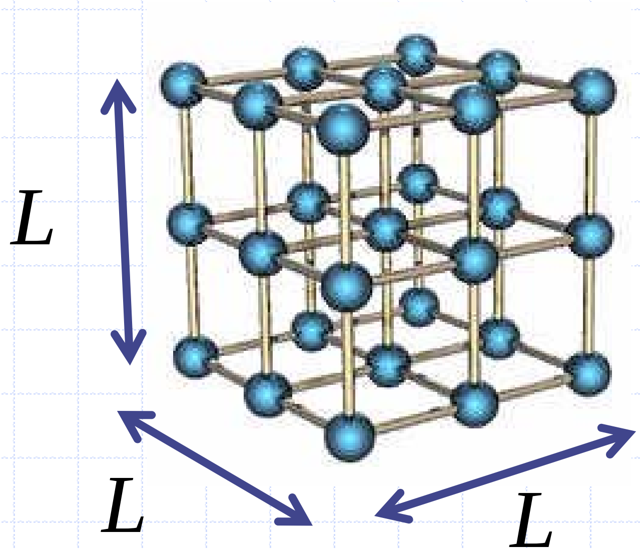




Ref: Wiki

A thermodynamic quantum many-body system

A physical system remains in its instantaneous eigenstate if a given perturbation is acting on it **slowly enough** and if there is a gap between the eigenvalue and the rest of the Hamiltonian's spectrum.



$$N = L^d$$

Thermodynamic limit

$$L \to \infty$$

Quantum adiabatic condition

Time-dependent Hamiltonian

$$H(t) = H_0 + H_I(t)$$

$$H(t)|\phi_n(t)\rangle = \epsilon_n(t)|\phi_n(t)\rangle$$

Quantum state and Schodinger Eq.

$$|\Psi(t)\rangle = \sum_n a_n(t)|\phi_n(t)\rangle \qquad i\hbar\frac{\partial}{\partial t}|\Psi(t)\rangle = H(t)|\Psi(t)\rangle$$

$$i\hbar\sum_n \left[\dot{a}_n(t)|\phi_n(t)\rangle + a_n(t)|\partial_t\phi_n(t)\rangle\right] = \sum_n a_n(t)\epsilon_n|\phi_n(t)\rangle$$

Hamiltonian

Unitary transformation

$$a_n(t) = \tilde{a}_n(t) \exp\left(-i \int^t \epsilon_n t' dt'\right)$$

$$\frac{\partial \tilde{a}_m}{\partial t} = -\tilde{a}_m \langle \phi_m | \partial_t \phi_m \rangle - \sum_{n \neq m} \frac{\langle \phi_m | \partial_t H | \phi_n \rangle \tilde{a}_n}{\omega_{nm}} \exp\left(-i \int^t \omega_{nm} dt'\right)$$

Hamiltonian

Time-dependent Hamiltonian

$$H(\lambda) = H_0 + \lambda H_I$$

$$\lambda = \lambda(x)$$

$$x = t/\tau_0$$

τ_0 is the duration time, for instance $\lambda = \dfrac{t}{\tau_0}$

$$\lambda \in [0,1] \quad \Rightarrow \quad t \in [0, \tau_0]$$

Hamiltonian

Time-dependent perturbation theory

$$\lambda \to \lambda + \delta\lambda \text{ from } t \to t + \Delta t$$

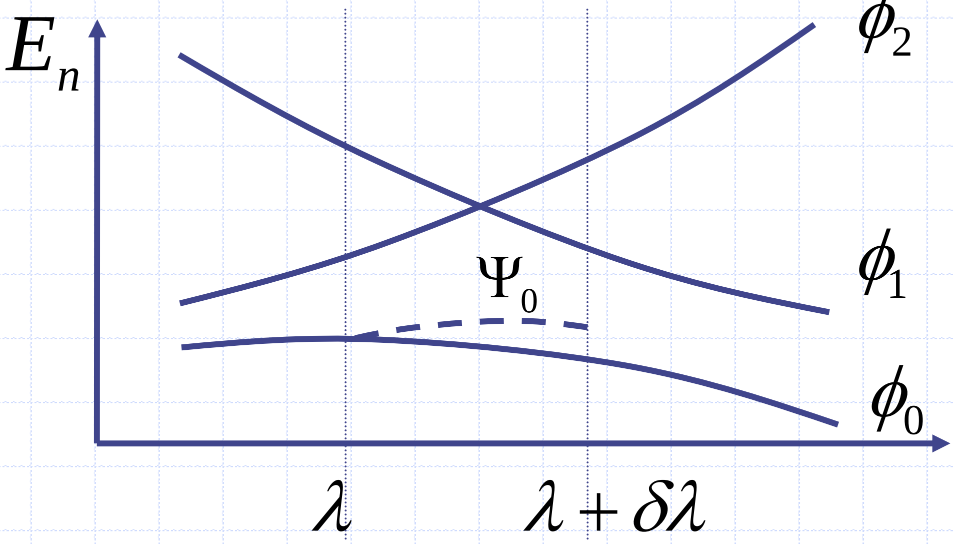
From initial conditions $\tilde{a}_0 = 1, \tilde{a}_m = 0$

$$\tilde{a}_0 \simeq 1 - \frac{\Delta t}{\tau_0}\lambda' \langle \phi_0 | \partial_\lambda \phi_0 \rangle$$

$$\tilde{a}_m(\Delta t) = -\int_0^{\Delta t} \frac{\langle \phi_m | \partial_t H | \phi_0 \rangle}{\epsilon_0 - \epsilon_m} e^{-\frac{i}{\hbar}\int^t [\epsilon_0 - \epsilon_m] dt'} dt$$

$$= -\frac{1}{\tau_0}\frac{d\lambda}{dx}\frac{\langle \phi_m | \partial_\lambda H | \phi_0 \rangle}{\epsilon_0 - \epsilon_m}\left[e^{-i[\epsilon_0 - \epsilon_m]\Delta t} - 1\right]$$

Hamiltonian



$$\Delta t >> \omega_{n0}$$

$$F = 1 - \frac{(\delta\lambda)^2}{2}\tilde{\chi}_F$$

$$\tilde{\chi}_F = \sum_{n\neq 0} \frac{|H_I^{n0}|^2}{(\epsilon_0 - \epsilon_n)^2}$$

$$\begin{aligned} F &= |\langle \phi_0(t)|\Psi(t)\rangle| \\ &\simeq 1 - \left(\frac{\lambda'}{\tau_0}\right)^2 \sum_{n\neq 0} \frac{|H_I^{n0}|^2[1-\cos(\omega_{0n}\Delta t)]}{\omega_{0n}^2} \end{aligned}$$

Hamiltonian

Extennd to a finite change of λ

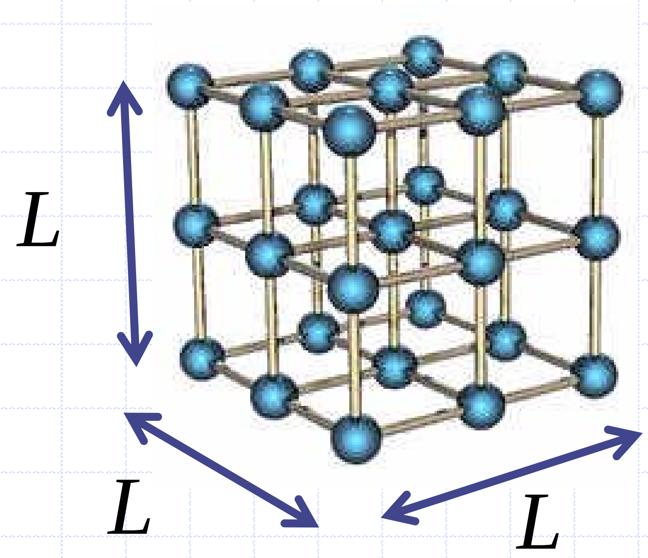
$$\lambda_i \rightarrow \lambda_f$$

How to define smallness of $\delta\lambda$

$$\delta\lambda = \frac{\lambda_f - \lambda_i}{M}$$

$$F = 1 - \frac{(\delta\lambda)^2}{2}\tilde{\chi}_F \qquad \tilde{\chi}_F \propto L^{d_a}$$

$$P \approx M\left(\frac{1}{M}\right)^2 L^{d_a} \quad \Rightarrow \quad M = L^{d_a}$$



$$N = L^d$$

Gu and Lin, arXiv: 08073491

Hamiltonian

The probability of staying in the ground state:

$$P \simeq \left[1 - \frac{1}{2}\left(\frac{\lambda'}{\tau_0}\right)^2 \tilde{\chi}_F\right]^{L^{d_a}}$$

The quantum adiabatic condition

$$|\lambda'| L^{d_a} \ll \tau_0$$

For linear quench

$$L^{d_a} \ll \tau_0$$

Discussion

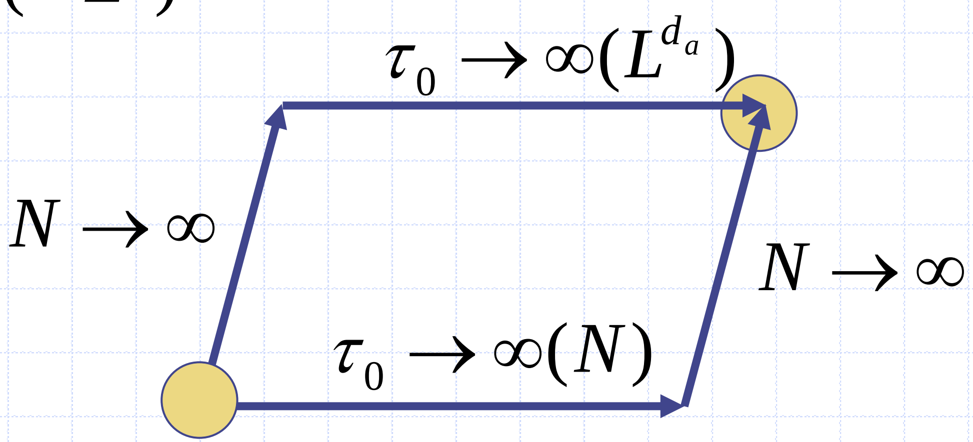
The quantum adiabatic condition

$$|\lambda'|L^{d_a} \ll \tau_0 \qquad N(=L^3) = 6.02\times 10^{23}$$

A physical acceptable duration time

$$\tau_0 \approx N(=L^d)$$

If, $d_a > d$



Then the quantum adiabatic theorem breaks down

Discussion

d_a=1,

Can you wait me

3 years ~ 10^8 s (~N)

Yes, I can.

But if d_a=2,

So can you want

180*3=540 years?

Of course, I can't.

Summary

1. We establish a general relation between the fidelity and dynamic structure factor of the driving parameter
2. We can learn the universality class of the critical phenomena from the fidelity susceptibility.
3. We derive a quantum adiabatic condition for quantum many-body systems in the thermodynamic limit.

謝謝大家

Thank you