The Quantum Spin Hall Effect

Shou-Cheng Zhang
Stanford University
with Andrei Bernevig, Taylor Hughes

Science*, 314,*1757 (2006)

Molenkamp et al,
Science*, 318, *766 (2007)

XL Qi, T. Hughes, SCZ
Nature Physics*, 4, *273 (2008)

The search for new states of matter

The search for new elements led to a golden age of chemistry.

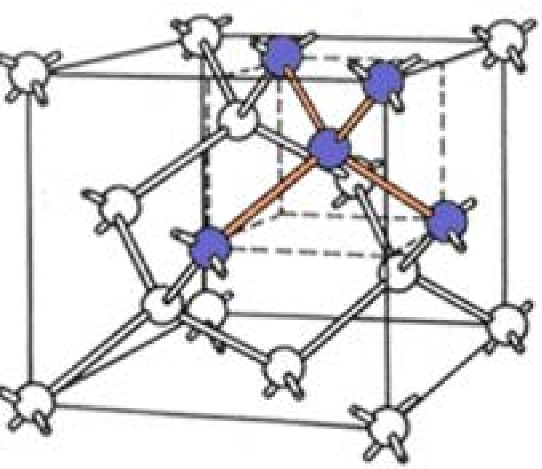
The search for new particles led to the golden age of particle physics.

In condensed matter physics, we ask what are the fundamental states of matter?

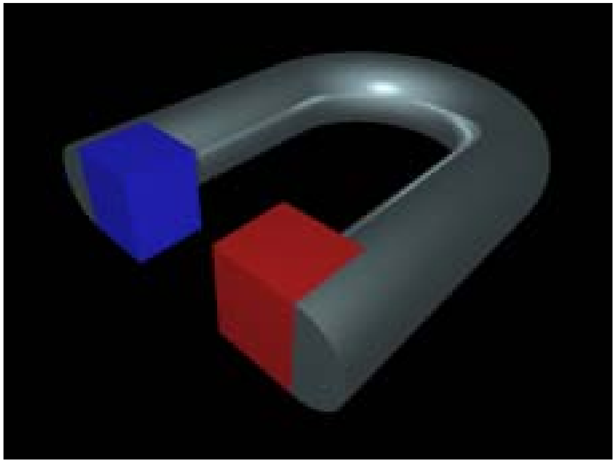
In the classical world we have solid, liquid and gas. The same H_2O molecules can condense into ice, water or vapor.

In the quantum world we have metals, insulators, superconductors, magnets etc.

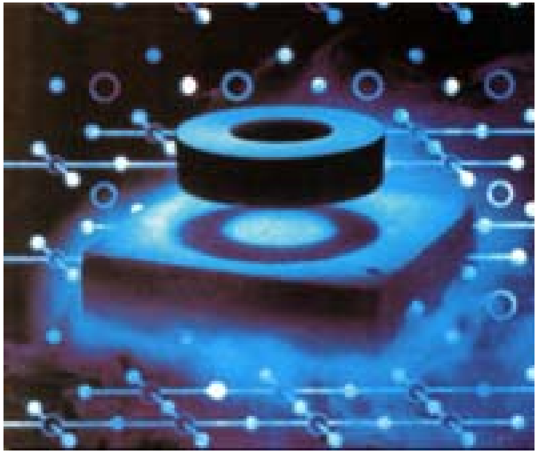
Most of these states are differentiated by the broken symmetry.



Crystal: Broken translational symmetry



Magnet: Broken rotational symmetry



Superconductor: Broken gauge symmetry

The quantum Hall state, a topologically non-trivial state of matter

$$\sigma_{xy} = n\frac{e^2}{h}$$

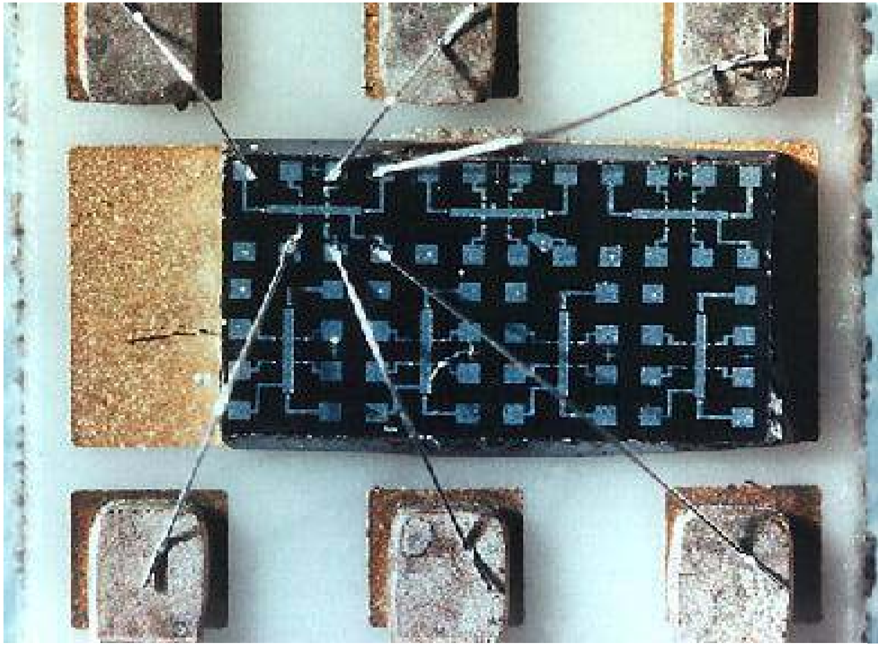
• Following Laughlin's gauge argument, TKNN showed that n is a topological integer, called the first Chern number.

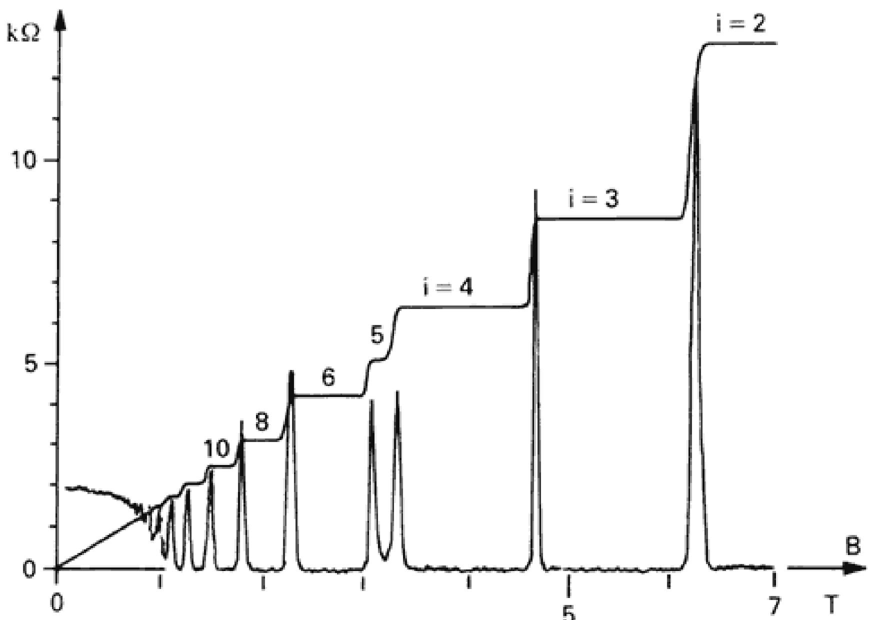
$$n = \int \frac{d^2k}{(2\pi)^2}\varepsilon^{\mu\nu}F_{\mu\nu}(k)$$

• A simple example of a topological integer:

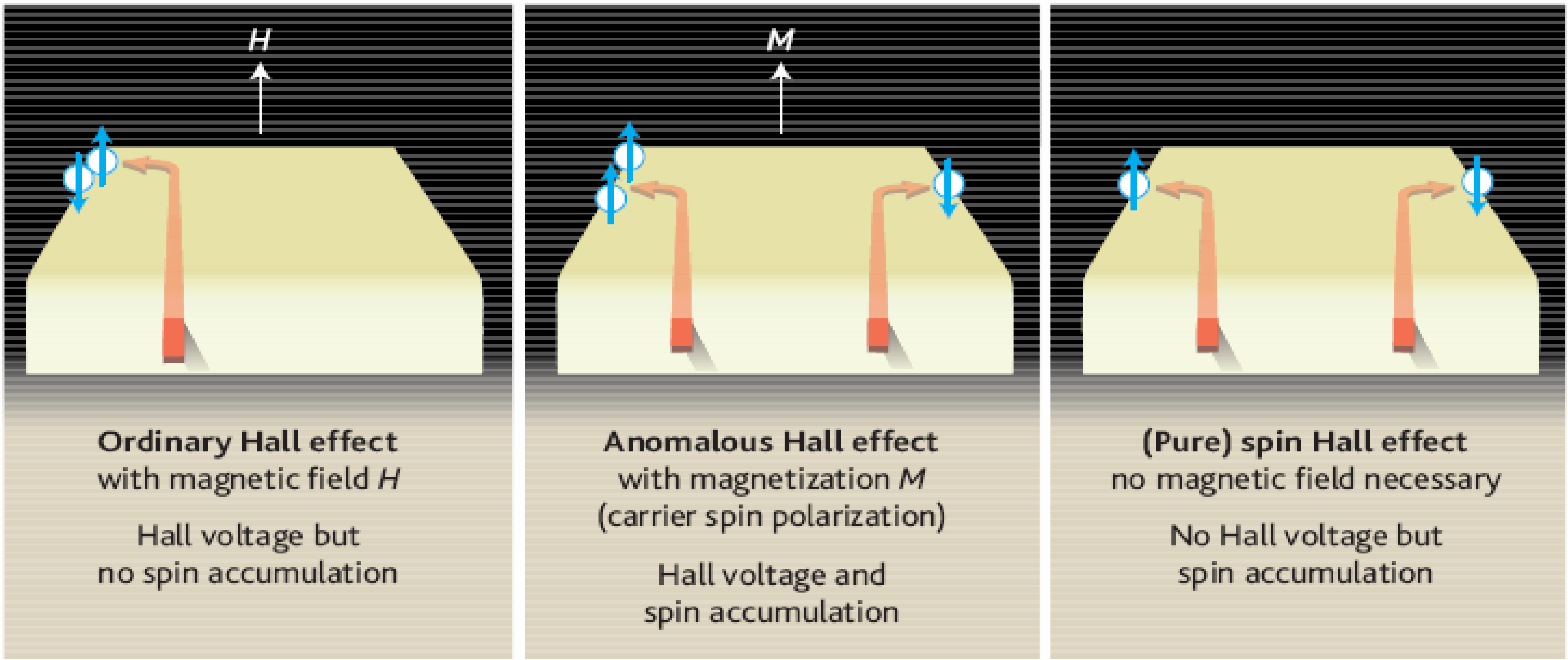
$$n = \int \frac{dx}{2\pi}\partial_x\theta(x)$$

$$e^{i\theta(x)} = 1,\ x = 0, 2\pi$$





The Generalizations of the Hall Effect



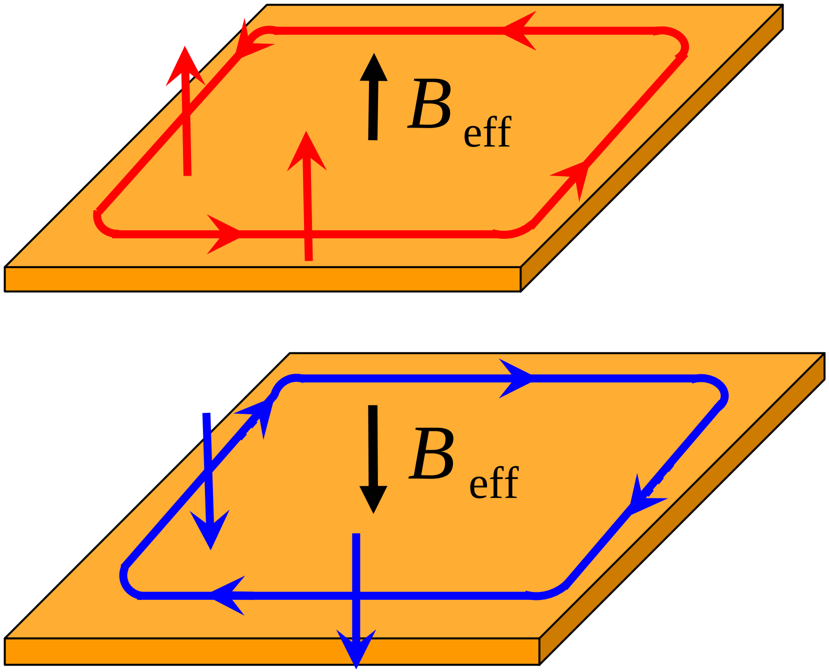
- Theoretical predictions of the intrinsic spin Hall effect (Science 2003, PRL 2004).
- The spin Hall effect has now been experimentally observed. (Science 2004, PRL 2004)

What about the quantum spin Hall effect?

Quantum Spin Hall Effect

• The QSH state can be thought of as two copies of QH states, one for each spin component, each seeing the opposite magnetic field. (Bernevig and Zhang, PRL, 2006)

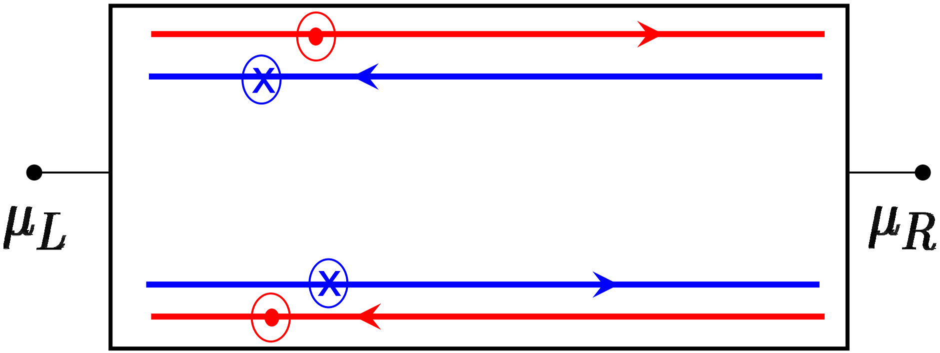
• The QSH state does not break the time reversal symmetry, and can exist without any external magnetic field.



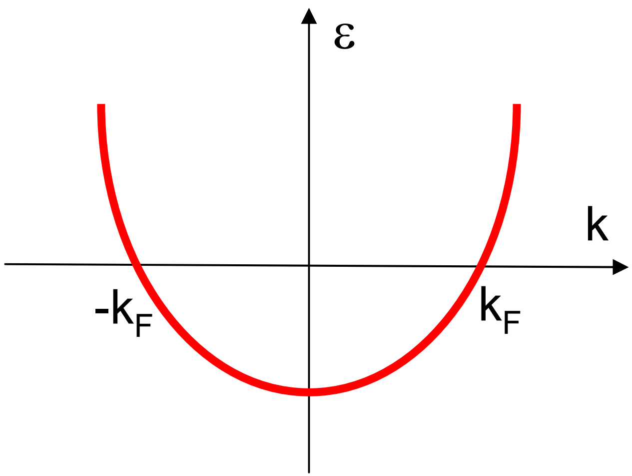
$$H_{so} = \lambda_{so}\vec{\sigma}(\vec{p} \times \vec{E})$$

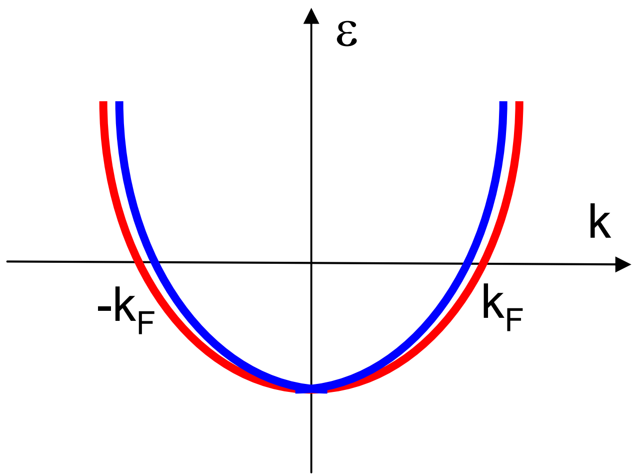
• Insulating gap in the bulk.

• Helical edge states: Two states with opposite spins counter-propagate at a given edge.



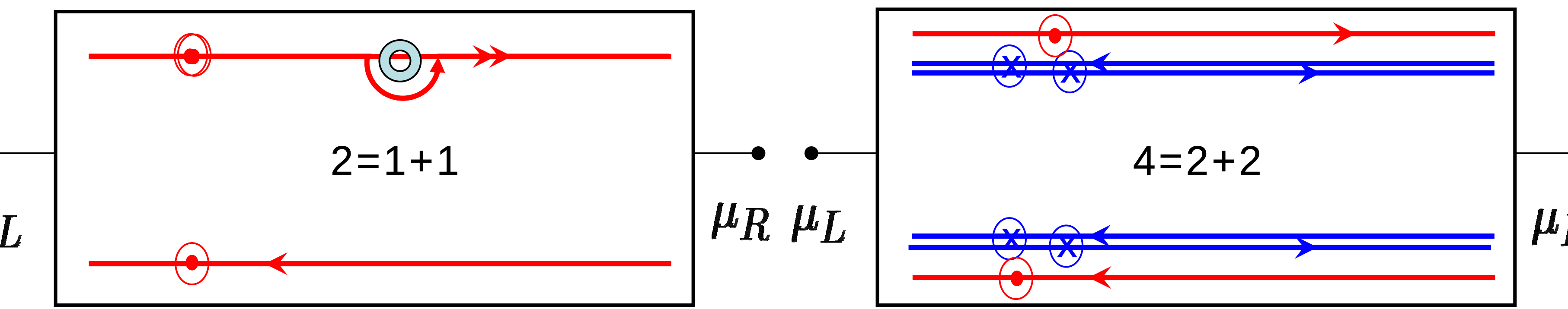
Chiral (QHE) and helical (QSHE) liquids in D=1





The QHE state spatially separates the two chiral states of a spinless 1D liquid

The QSHE state spatially separates the four chiral states of a spinful 1D liquid



No go theorems: chiral and helical states can never be constructed microscopically from a purely 1D model (Wu, Bernevig, Zhang, 2006; Nielsen, Ninomiya, 1981)

Taking the square root in math and physics

$$\sqrt{-1} = i$$

$$\sqrt{Klein\,Gordon} = Dirac$$

$$\sqrt{Dirac} = Chiral\;\; fermion$$

$$\sqrt{Space\,Time\,Symmetry} = Supersymmetry$$

$$\sqrt{Gravity} = Supergravity$$

$$\sqrt{1D\,spinless\,liquid} = Chiral\,edge\,state\,of\;\; QHE$$

$$\sqrt{1D\,spinful\,liquid} = Helical\,edge\,state\,of\;\; QSHE$$

Time reversal symmetry in quantum mechanics

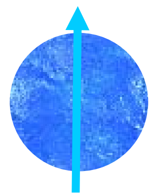
- Wave function of a particle with integer spin changes by 1 under 2π rotation.

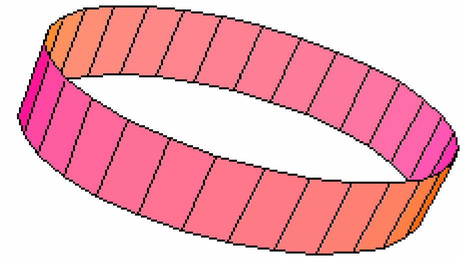
- Wave function of a half-integer spin changes by -1 under 2π rotation.

- Kramers theorem, in a time reversal invariant system with half-integer spins, $T^2=-1$, all states for degenerate doublets.

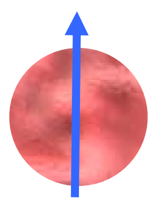
- Application in condensed matter physics: Anderson's theorem. BCS pair=(k,✸)+(-k,✵). General pairing between Kramers doublets.

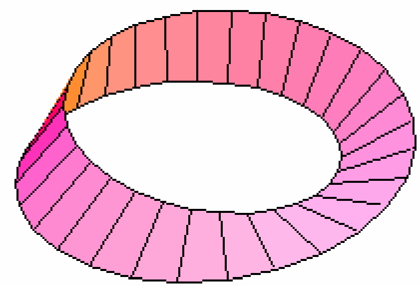
Spin=1





Spin=1/2

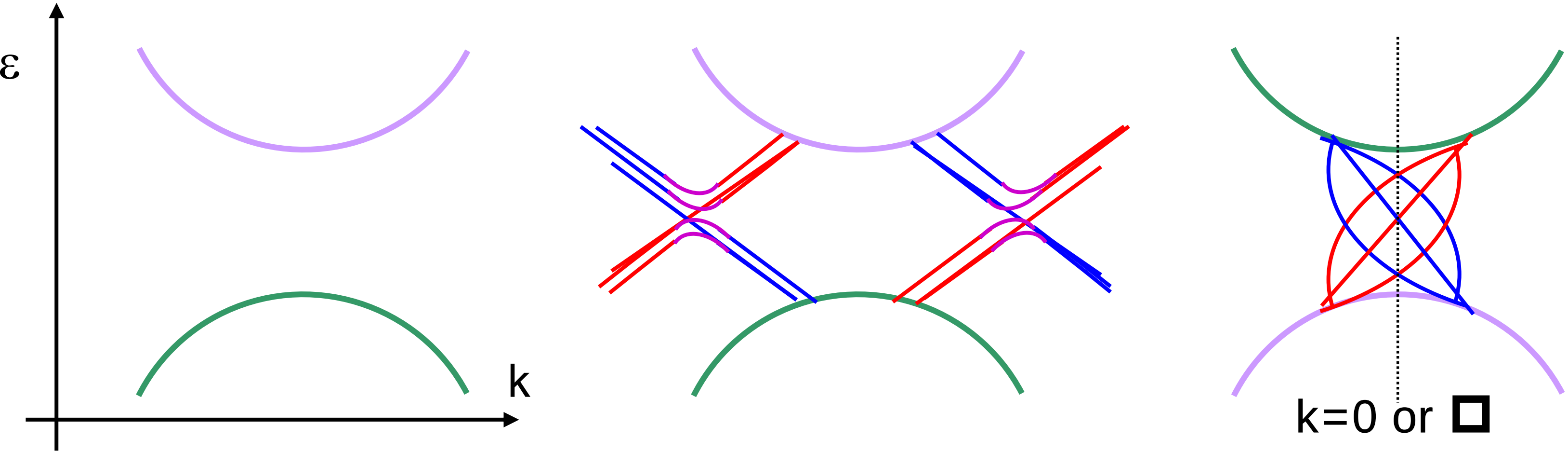




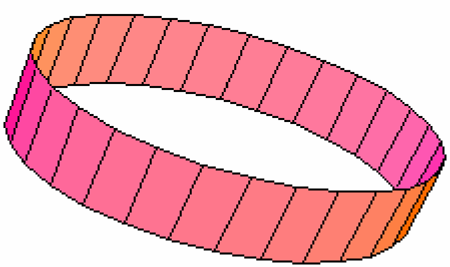
The topological distinction between a conventional insulator and a QSH insulator

Kane and Mele PRL, (2005); Wu, Bernevig and Zhang, PRL (2006); Xu and Moore, PRB (2006)

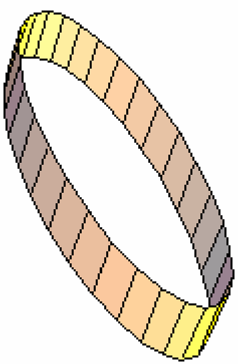
• Band diagram of a conventional insulator, a conventional insulator with accidental surface states (with animation), a QSH insulator (with animation). Blue and red color code for up and down spins.



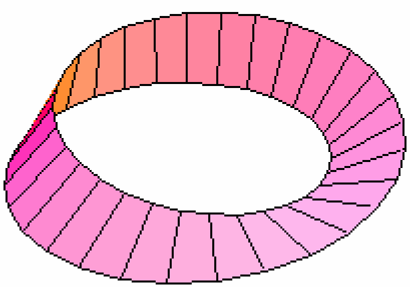
Trivial



Trivial



Non-trivial



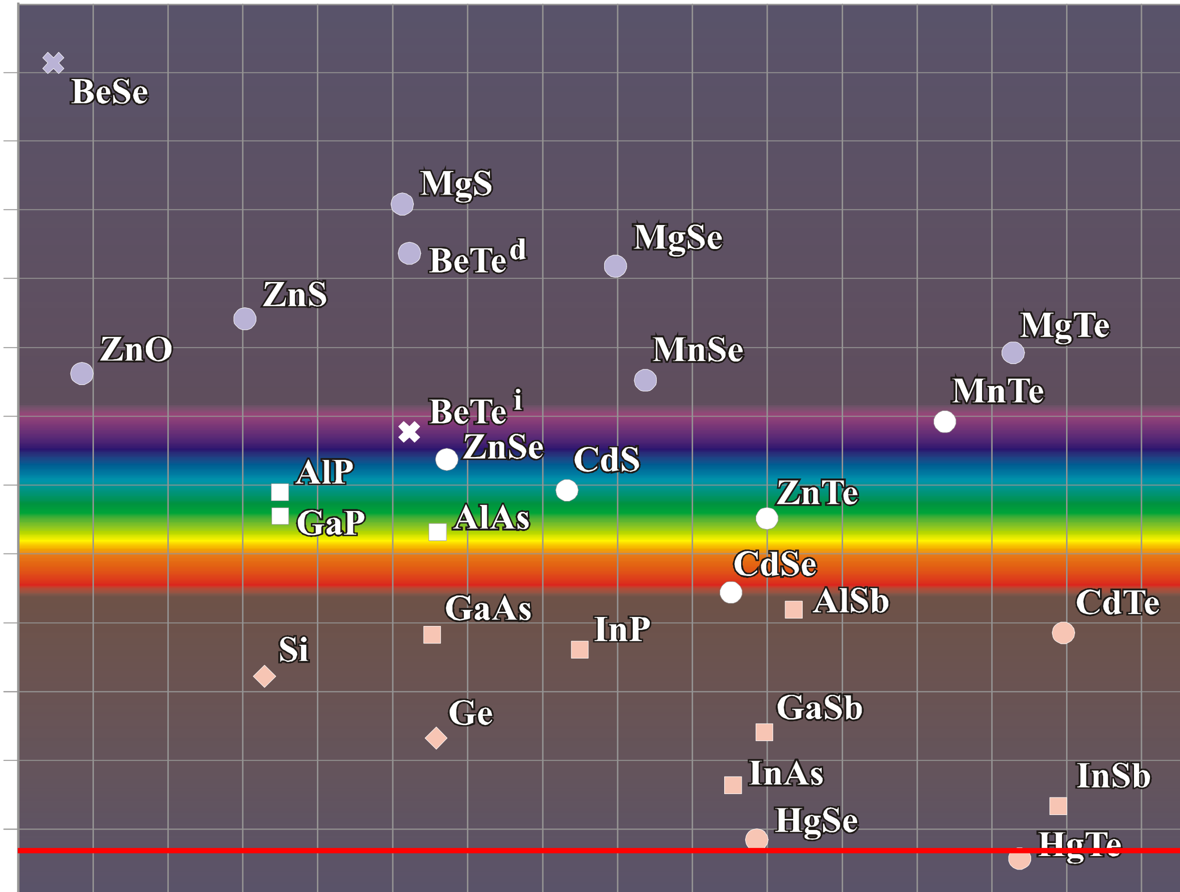
From topology to chemistry: the search for the QSH state

• Graphene – spin-orbit coupling only about 10^{-3}meV. Not realizable in experiments. (Kane and Mele, 2005, Yao et al, 2006, MacDonald group 2006)

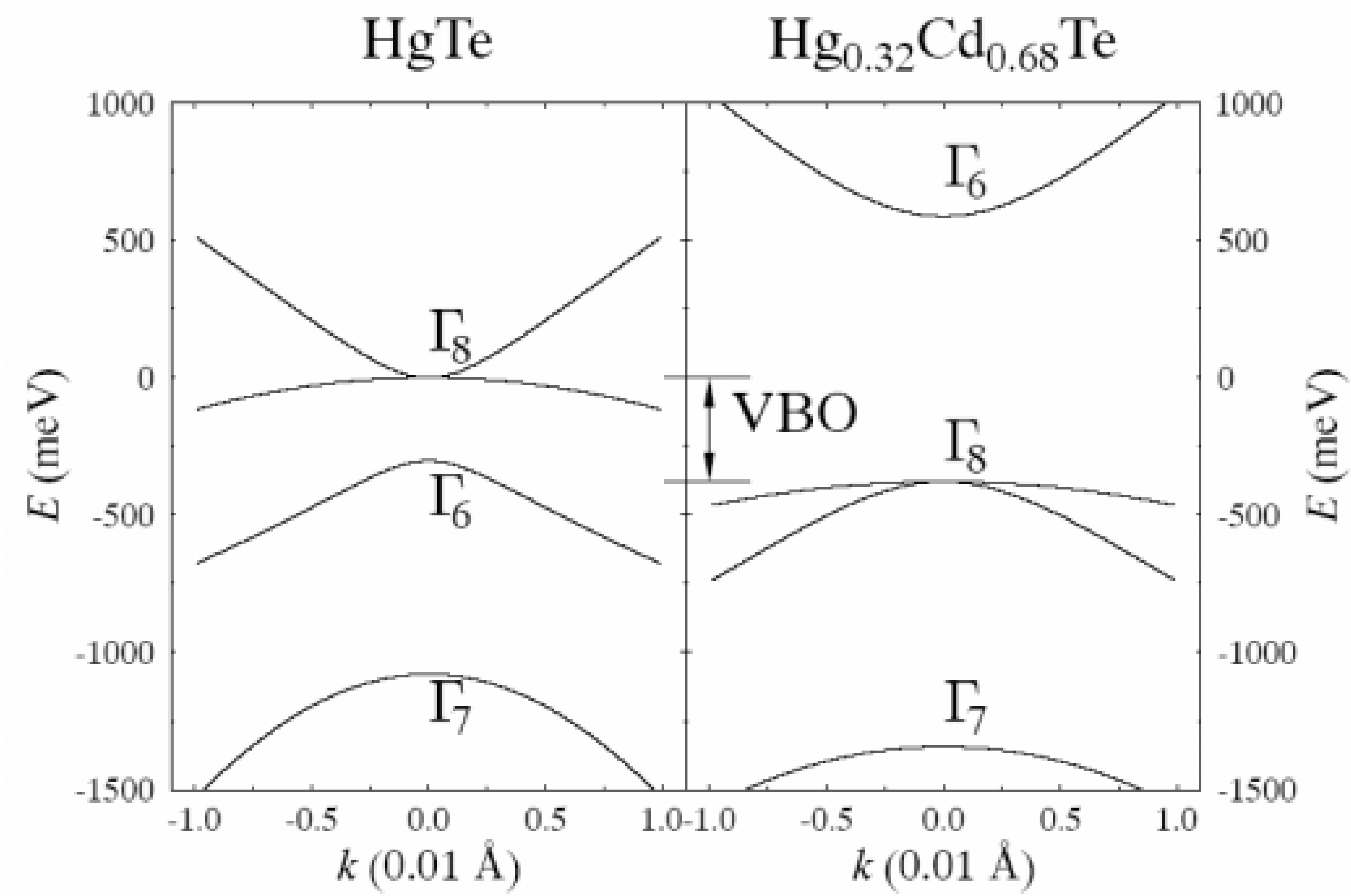
• Quantum spin Hall with Landau levels – spin-orbit coupling in GaAs too small. (Bernevig and Zhang, PRL, 2006)

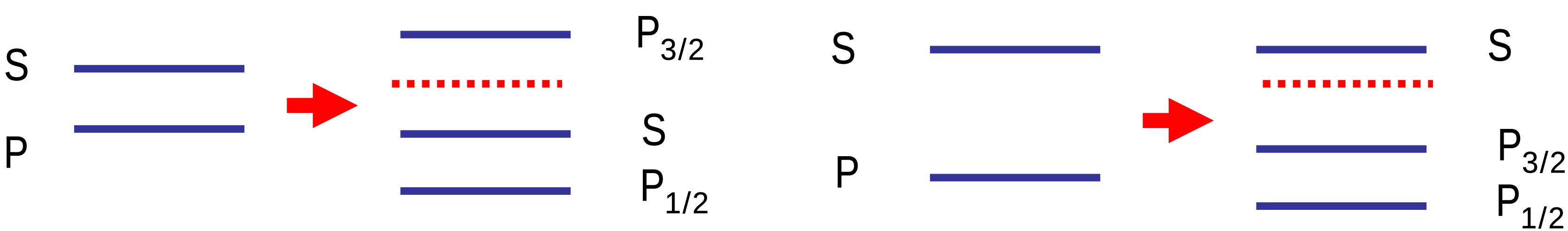
• Type III quantum wells work. HgTe has a negative band gap! (Bernevig, Hughes and Zhang, Science 2006)

• Tuning the thickness of the HgTe/CdTe quantum well leads to a topological quantum phase transition into the QSH state.



Band Structure of HgTe

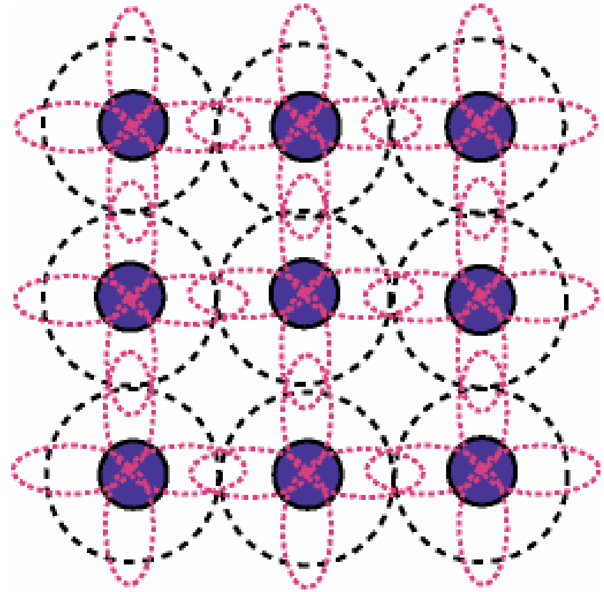




Effective tight-binding model

Square lattice with 4-orbitals per site:

$$|s,\uparrow\rangle, |s,\downarrow\rangle, |(p_x+ip_y,\uparrow\rangle, |-(p_x-ip_y),\downarrow\rangle$$



Nearest neighbor hopping integrals. Mixing matrix elements between the s and the p states must be odd in k.

$$H_{eff}(k_x,k_y)=\begin{pmatrix} h(k) & 0 \\ 0 & h^*(-k) \end{pmatrix}$$

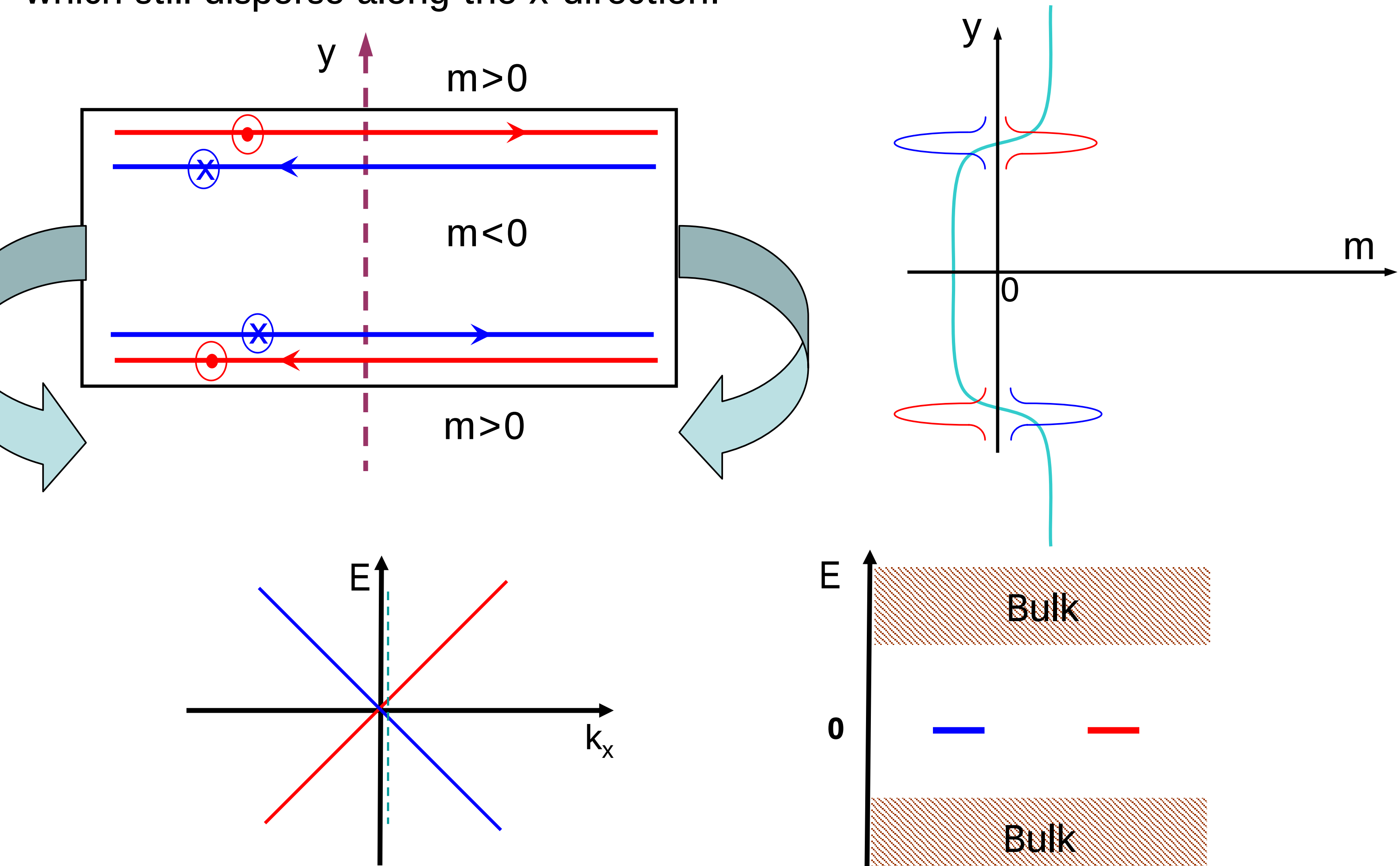
$$h(k)=\begin{pmatrix} m(k) & A(\sin k_x - i\sin k_y) \\ A(\sin k_x + i\sin k_y) & -m(k) \end{pmatrix} \equiv d_a(k)\tau^a$$

$$\Rightarrow \begin{pmatrix} m & A(k_x-ik_y) \\ A(k_x+ik_y) & -m \end{pmatrix} \qquad \Delta\sigma^{\uparrow}{}_{xy}=\frac{1}{2}\Delta\, sign(m) \qquad \Delta\sigma^{\downarrow}{}_{xy}=-\Delta\sigma^{\uparrow}{}_{xy}$$

Relativistic Dirac equation in 2+1 dimensions, with a tunable mass term!

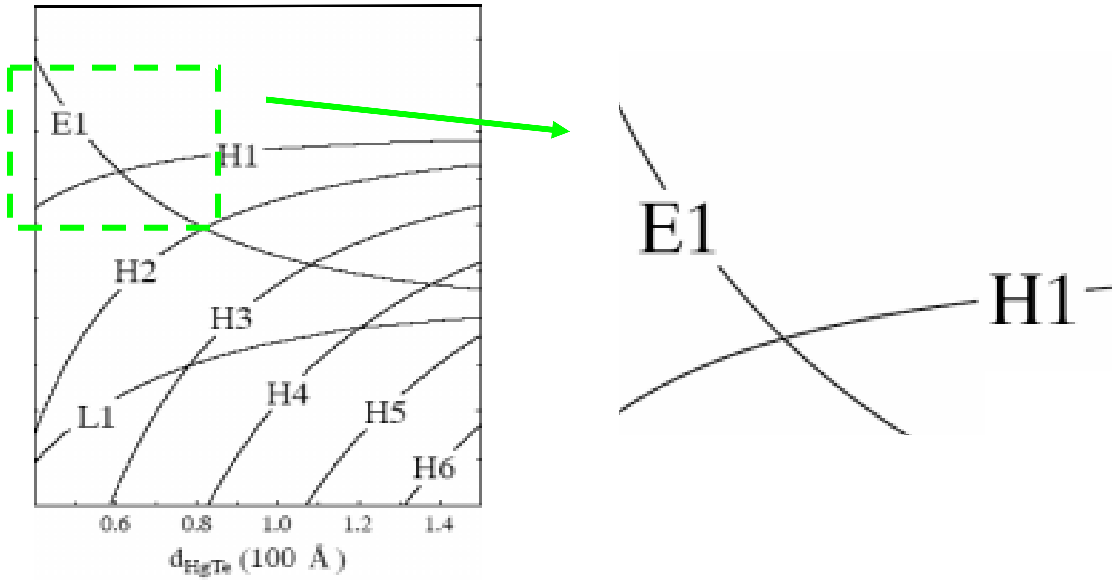
Mass domain wall

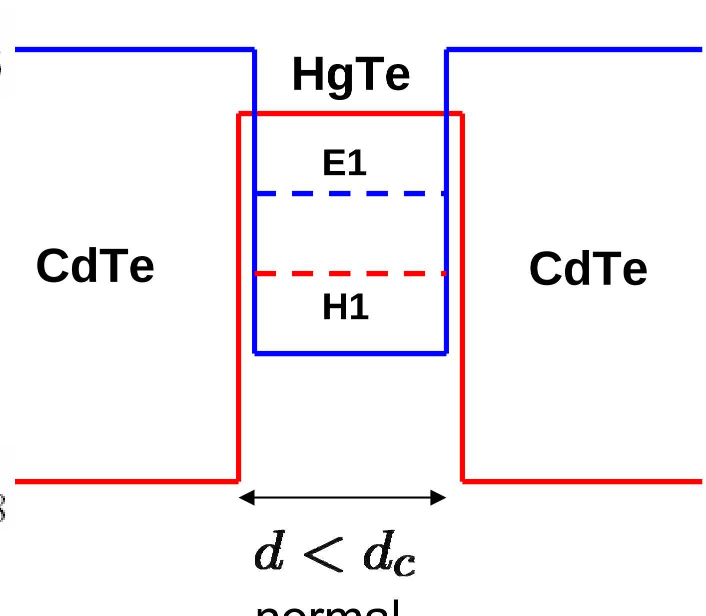
Cutting the Hall bar along the y-direction we see a domain-wall structure in the band structure mass term. This leads to states localized on the domain wall which still disperse along the x-direction.

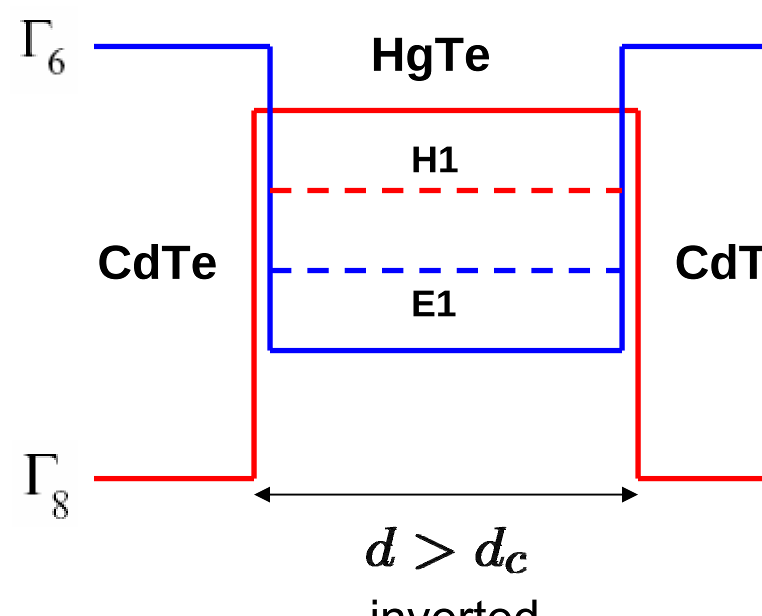


Quantum Well Sub-bands

Let us focus on E1, H1 bands close to crossing point

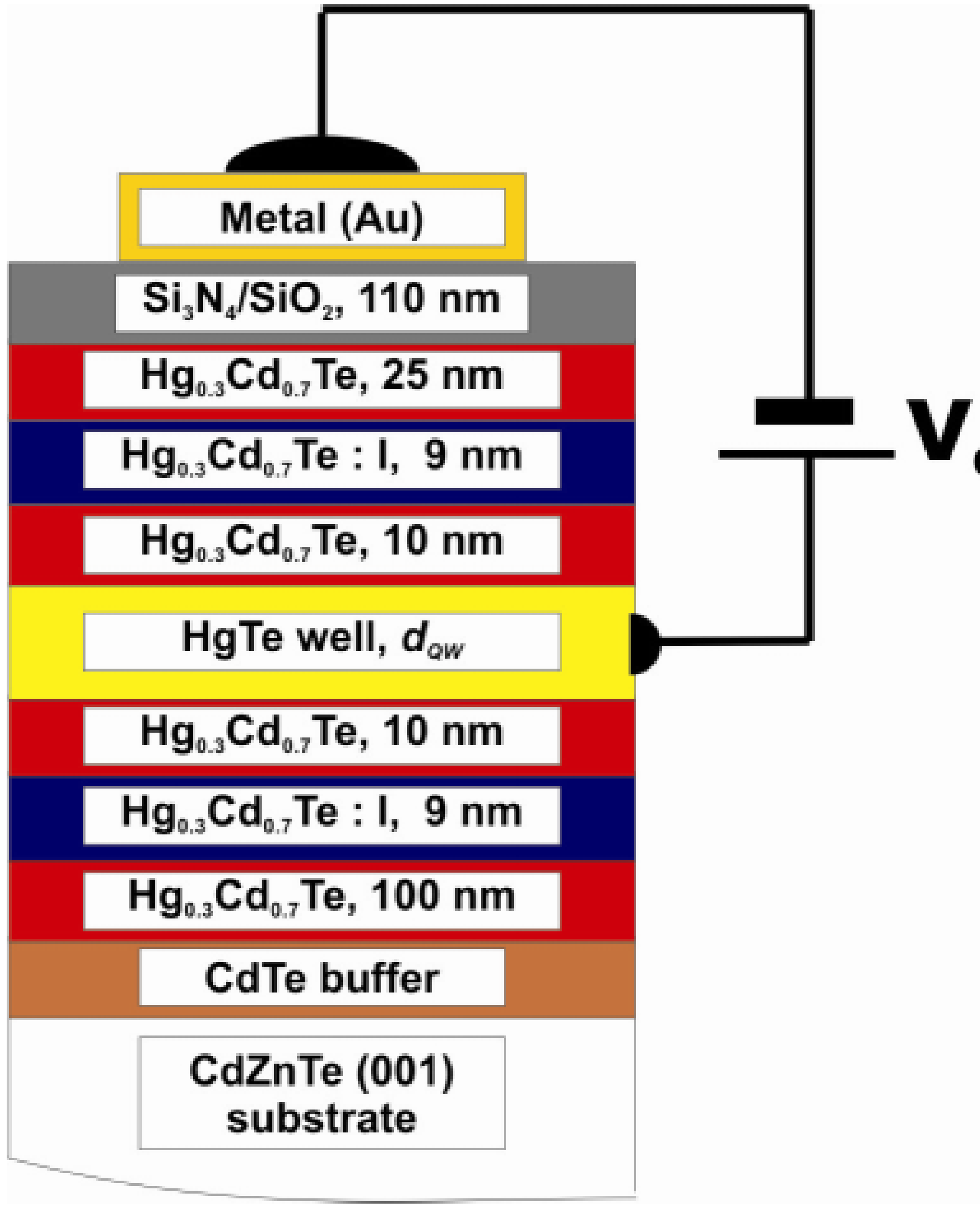




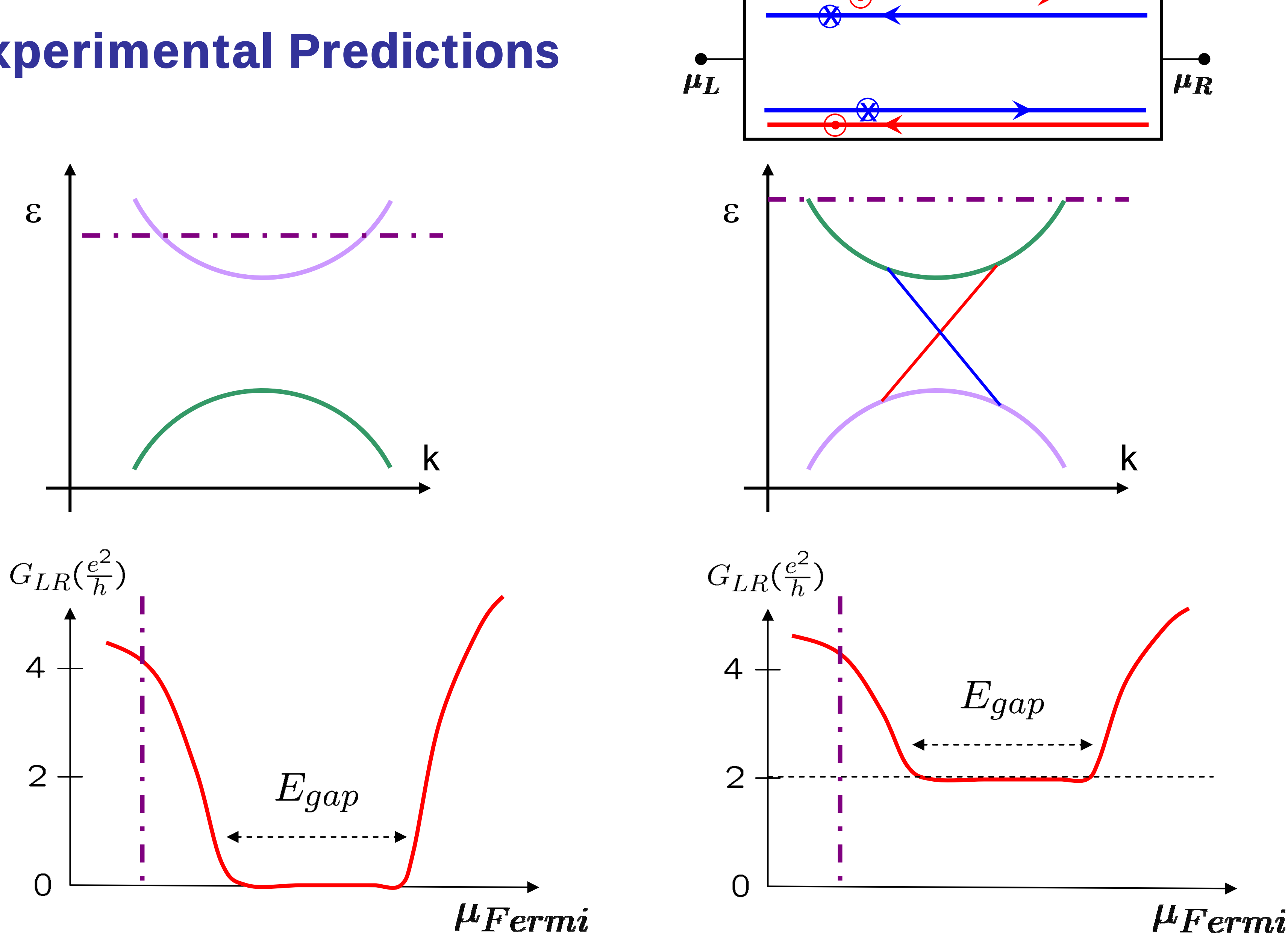


Experimental setup

• High mobility samples of HgTe/CdTe quantum wells have been fabricated.
• Because of the small band gap, about several meV, one can gate dope this system from n to p doped regimes.
• Two tuning parameters, the thickness d of the quantum well, and the gate voltage.



Experimental Predictions



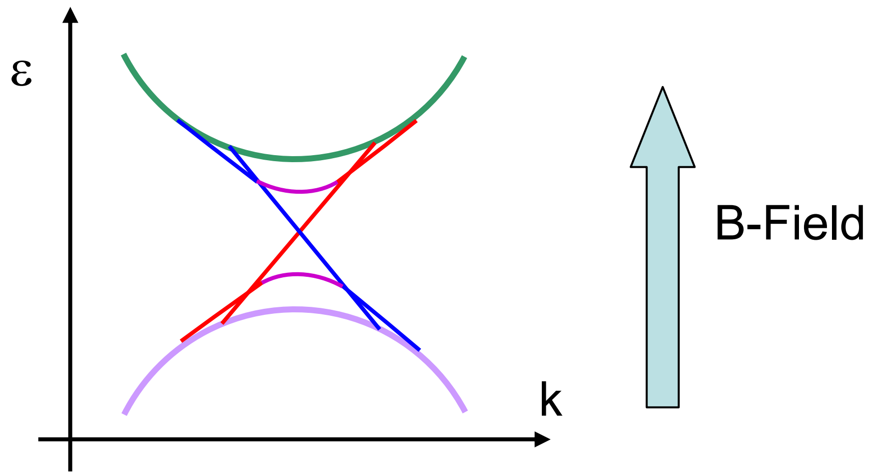
$d < d_c$, normal regime

$d > d_c$, inverted regime

Smoking gun for the helical edge state: Magneto-Conductance

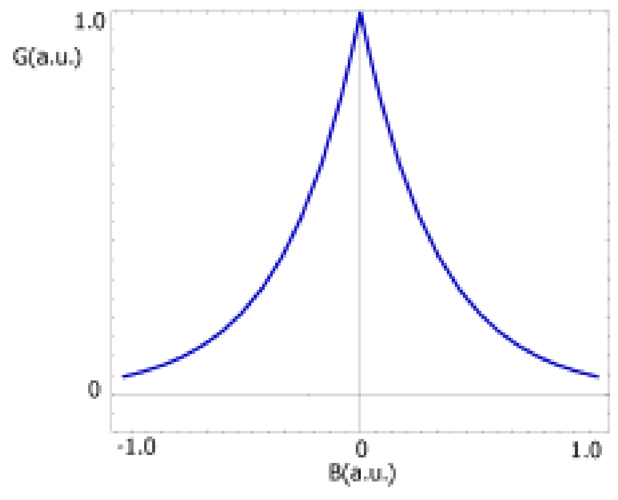
The crossing of the helical edge states is protected by the TR symmetry. TR breaking term such as the Zeeman magnetic field causes a singular perturbation and will open up a full insulating gap:

$$E_g \propto g|B|$$

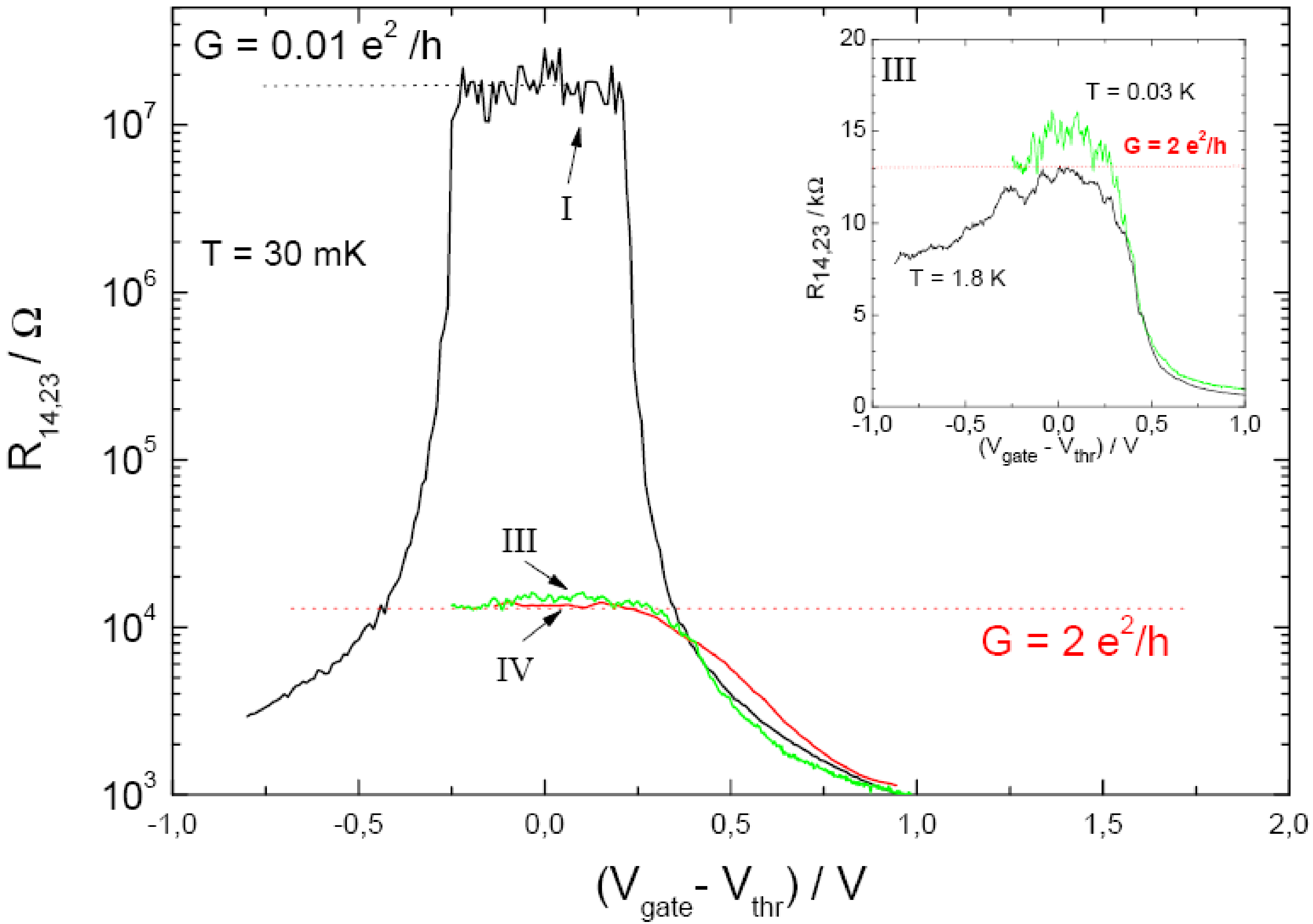


Conductance now takes the activated form:

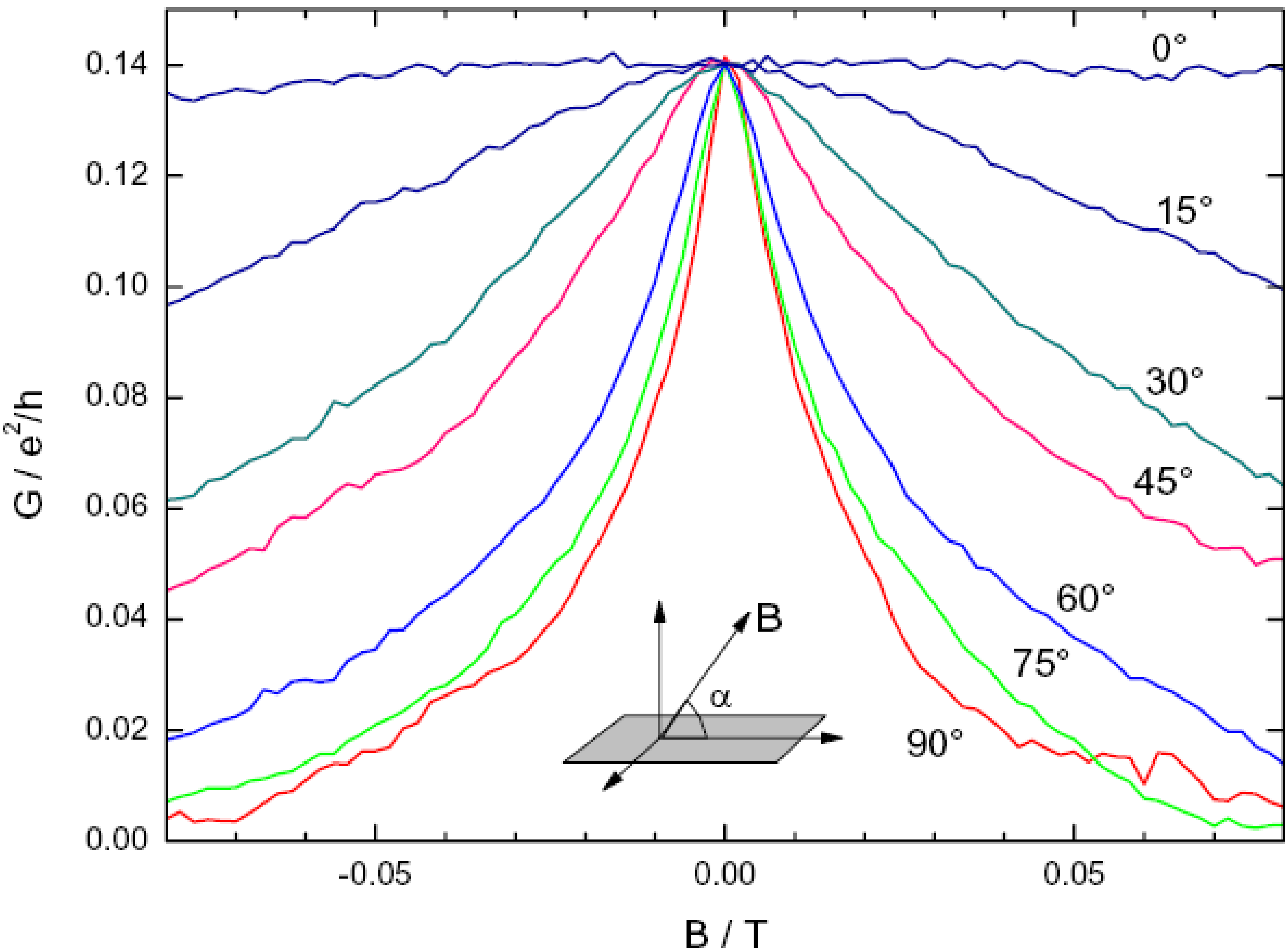
$$\sigma \propto f(T)e^{-g|B|/kT}$$



Experimental evidence for the QSH state in HgTe

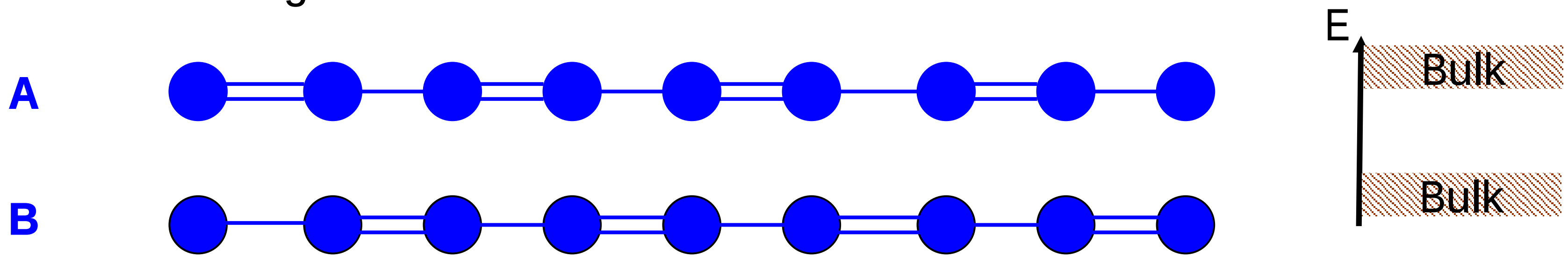


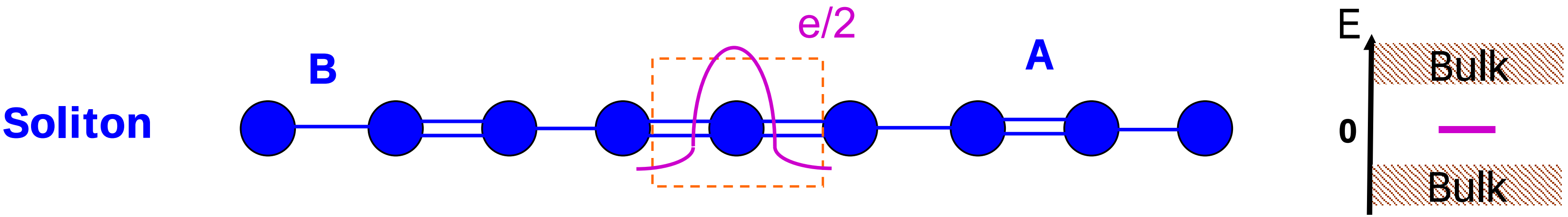
Magnetic field dependence of the residual conductance



A brief history of fractional charge

- Jackiw & Rebbi (PRD (1976)) predicted that a fractional charge e/2 is carried by the mass domain wall (soliton) of 1-d Dirac model.
- Su, Schrieffer and Heeger (PRB (1979)) presented a model of polyacetylene with two-fold degenerate ground states. A domain wall defect carries fractional charge e/2.

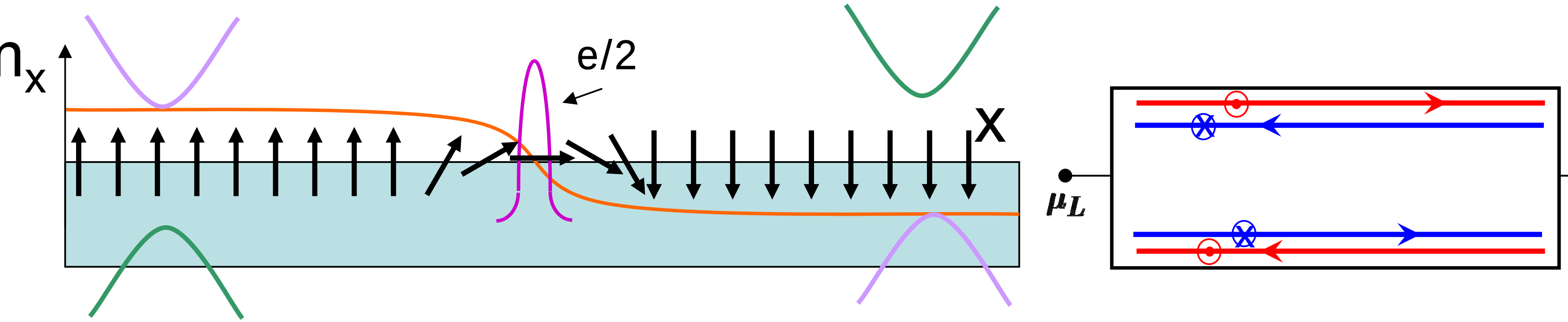




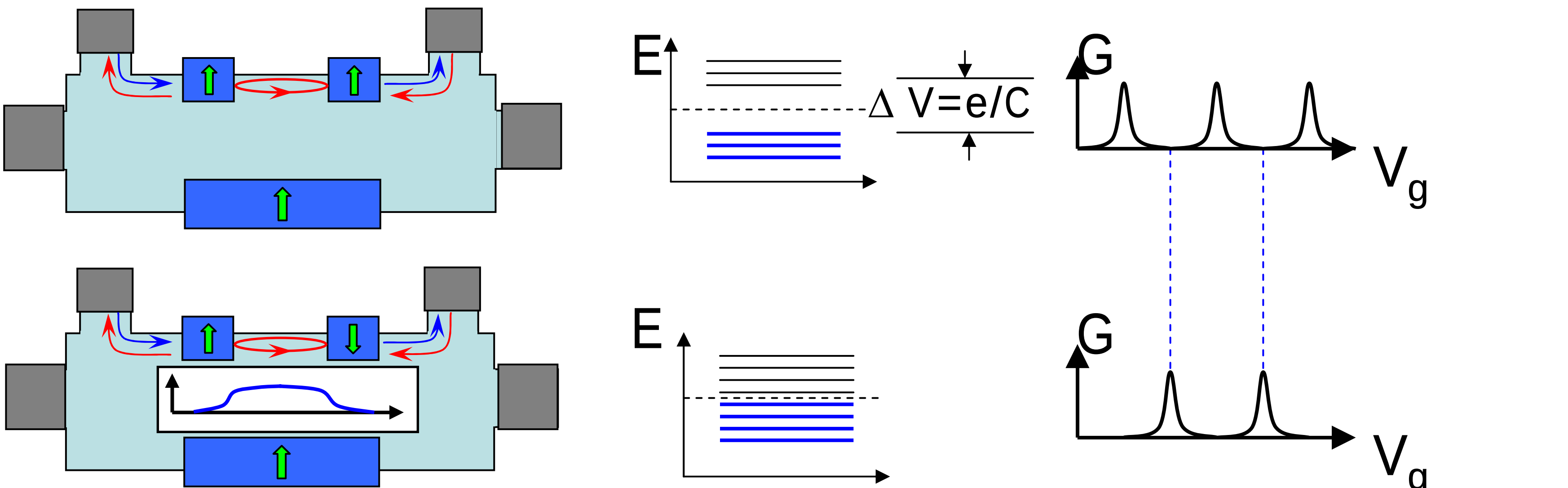
- Because of both up and down spin components carry fractional charge e/2, the net system only carries integer charge. Fractional charge has never been observed in any 1D system!

Fractional charge in the QSH state

- Since the mass is proportional to the magnetization, a magnetization domain wall leads to a mass domain wall on the edge.

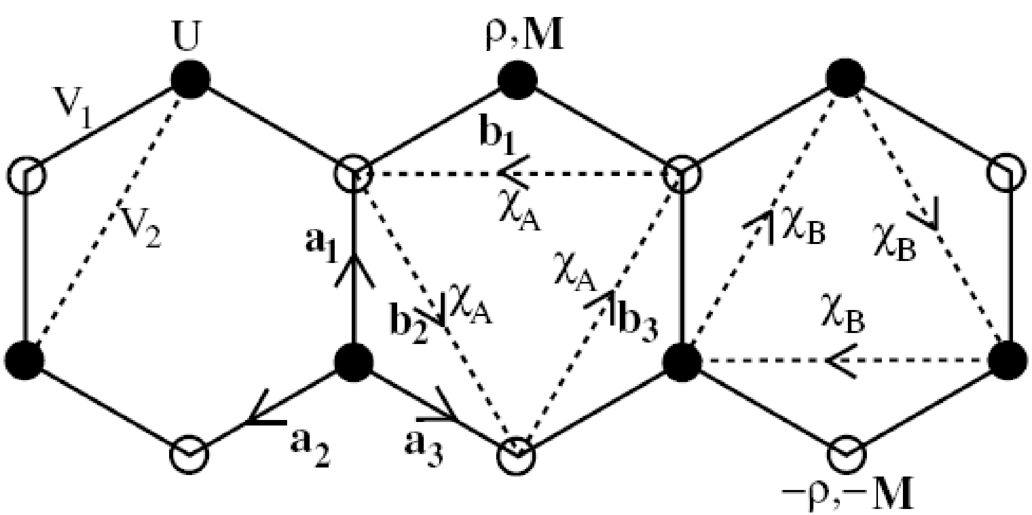


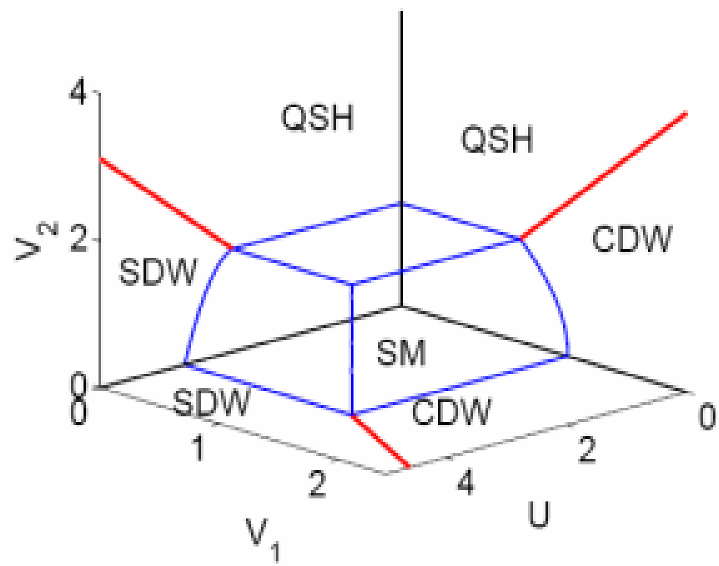
- The fractional charge e/2 can be measured by a Coulomb blockade experiment, one at the time!



Topological Mott insulators

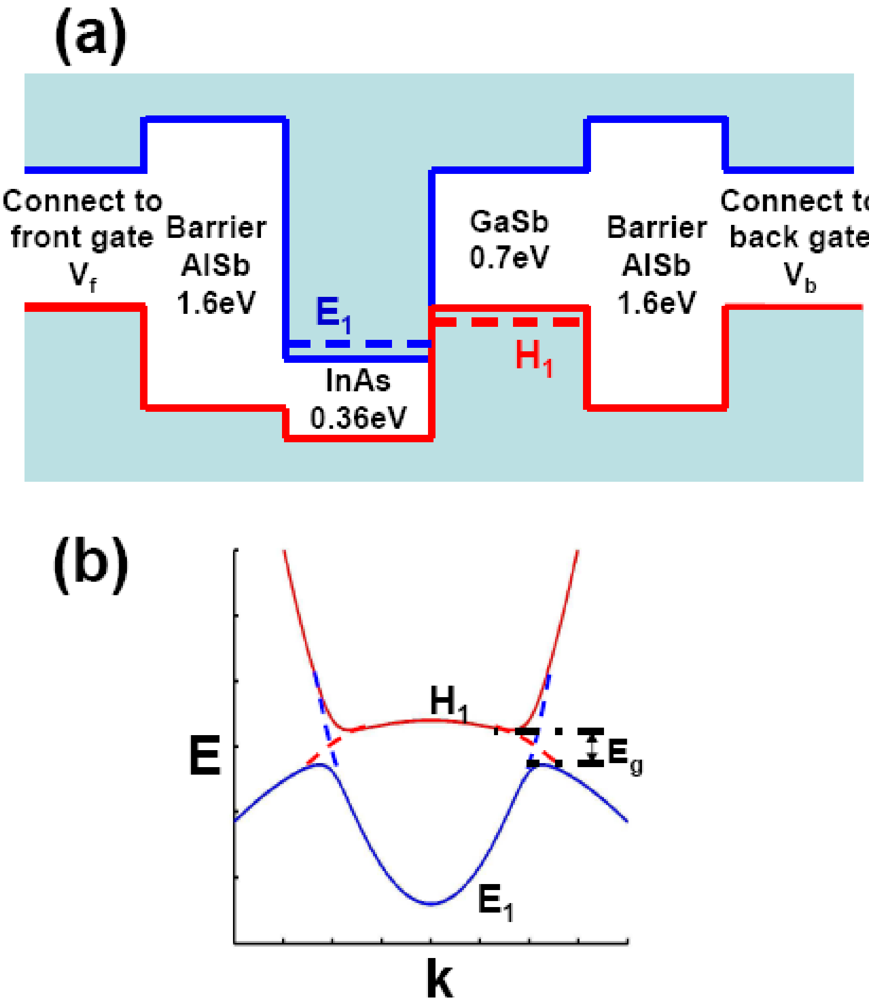
- So far, the QSH insulator is a topologically non-trivial band insulator. Can we have a topological Mott insulator, where the topologically non-trivial gap arises from interactions, not from band structure?
- Yes, on a honeycomb lattice with U, V1 and V2, one can obtain a TMI phase in the limit of V2>>U, V1. (Raghu et al, arXiv:0710.0030)
- This model provides an example of dynamic generation of spin-orbit coupling. (Wu+Zhang, PRL 2004).





QSH in InAs/GaSb quantum wells

• HgTe is not a material that can be easily fabricated. We are searching for new semiconductor materials which can lead to QSH.
• In HgTe, the band inversion occurs intrinsically in the material. However, in InAs/GaSb quantum wells, a similar inversion can occur, since the valance band edge of GaSb lies above the conduction band edge of InAs.
• Our theoretical work show that the QSH can occur in InAs/Gab quantum wells. This material can be fabricated commercially in many places around the world.

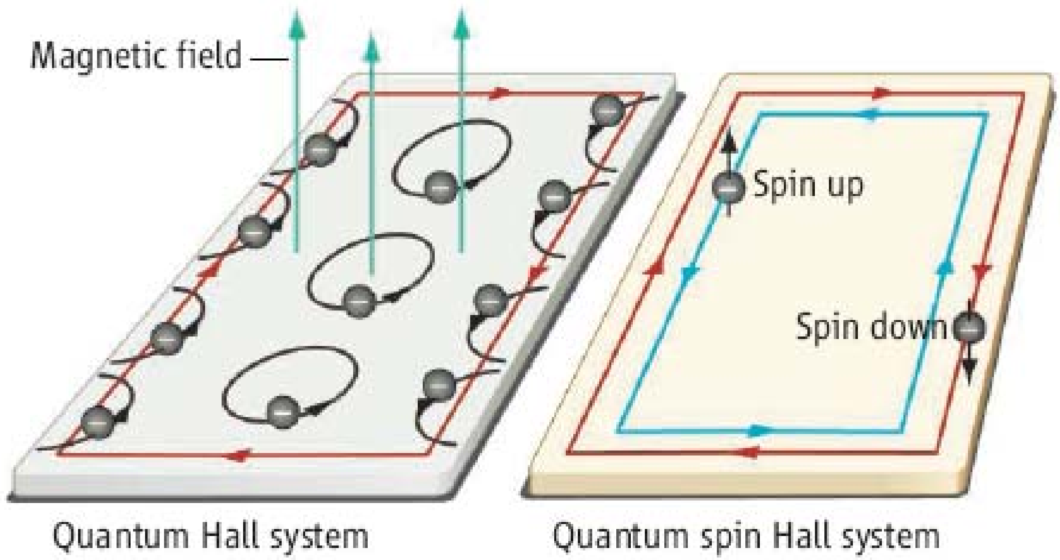


HYSICS

A New State of Quantum Matter

Experiments show that electron spins can flow without dissipation in a novel electrical insulator.

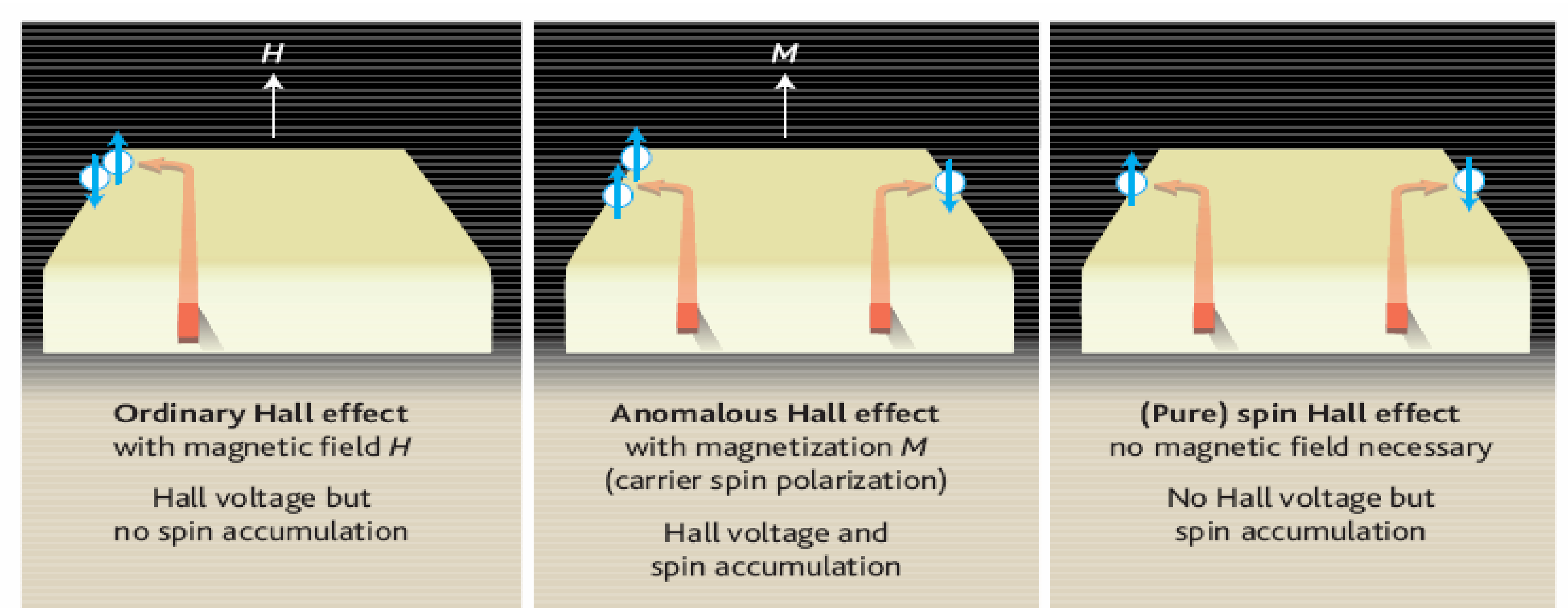
aoto Nagaosa



Quantum spin Hall effect shows up in a quantum well insulator, just as predicted

The effect, which occurs without a magnetic field, is a new and topologic ly distinct electronic state.

Completing the table of Hall effects

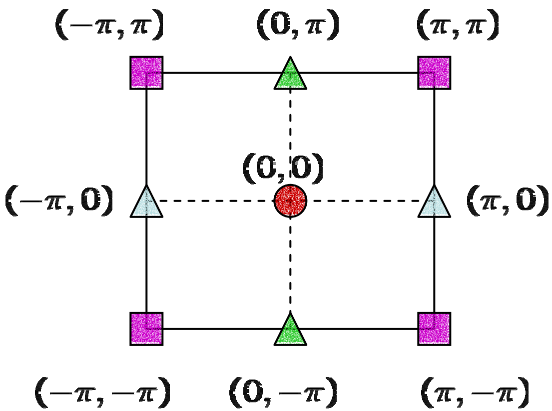


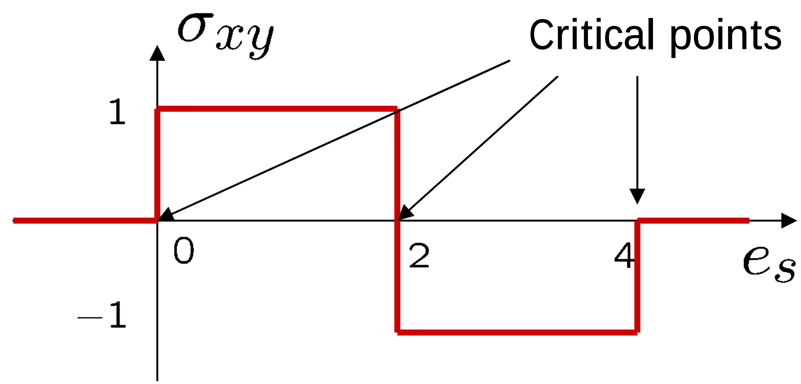
Hall **1879**	**Anomalous Hall** **1889**	**Spin Hall** **2004**
QHE **1980**	**QAHE** **2008?**	**QSHE** **2007**

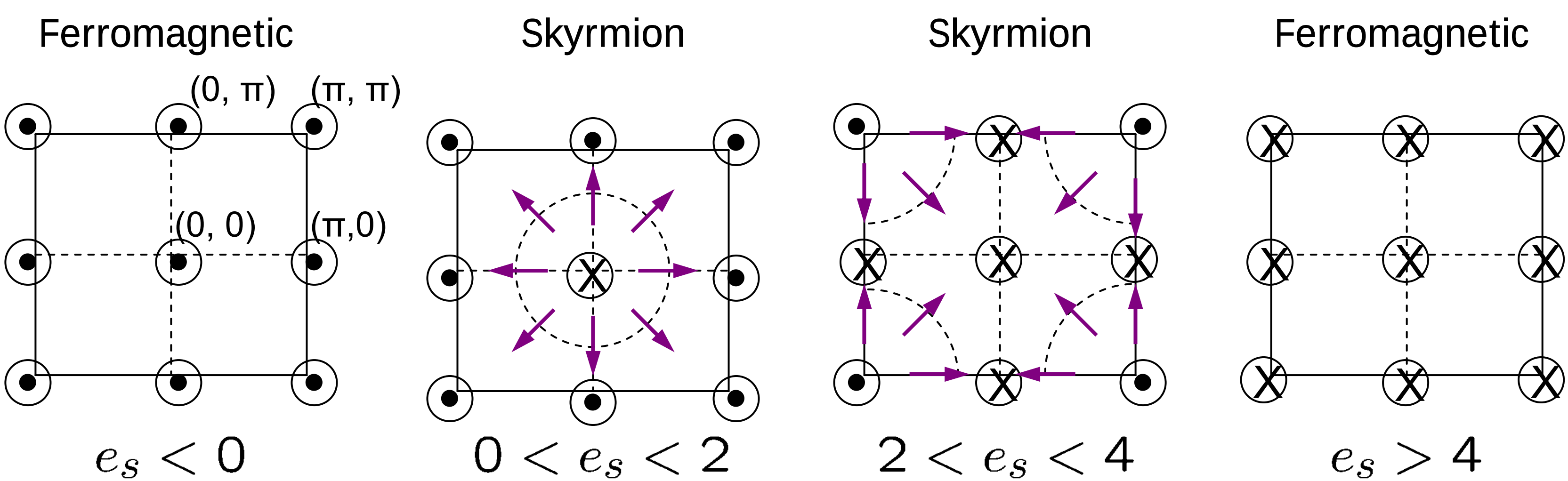
Momentum space topology of the tight-binding model

$$h(k) = d_a(k)\tau^a$$

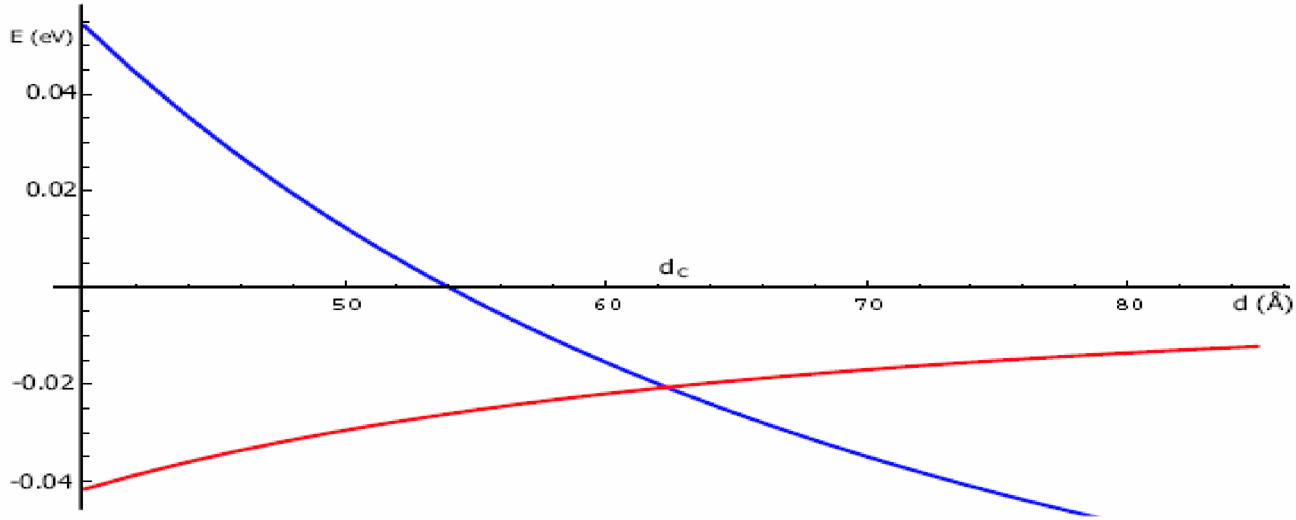
$$\sigma_{xy} = -\frac{1}{8\pi^2}\int\int dk_x dk_y \hat{\mathbf{d}} \cdot \partial_{\mathbf{x}}\hat{\mathbf{d}} \times \partial_{\mathbf{y}}\hat{\mathbf{d}}$$



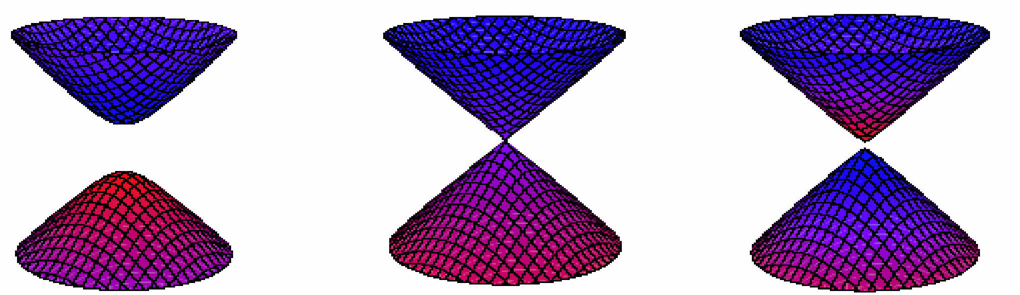


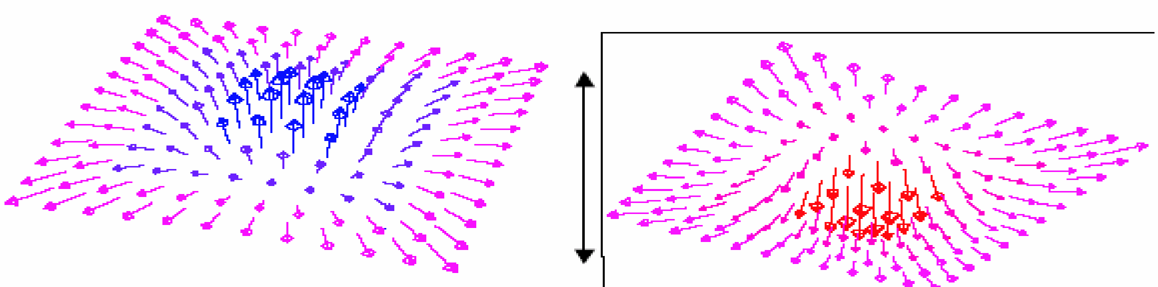


Topological quantum phase transition



Meron in continuum picture:





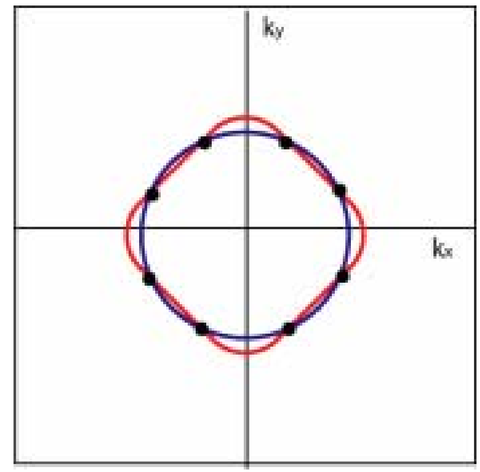
$$\Delta\sigma_{xy}^{(s)} = 2\frac{e^2}{h}$$

Inversion symmetry breaking in zincblend lattices

Inversion breaking term comes in the form:

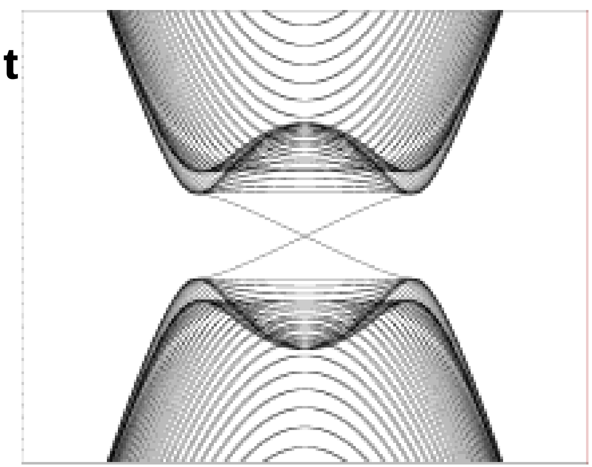
$$C(<k_z> + \ldots)\{J_z, J_x^2 - J_y^2\},$$ $J_x,\ J_y,\ J_z$ -spin 3/2 matrices

which couples E1+, H1- and E1-,H1+ states and is a constant in quasi-2d systems



$$H_\Delta^{eff} = \begin{pmatrix} 0 & 0 & 0 & -\Delta \\ 0 & 0 & \Delta & 0 \\ 0 & \Delta & 0 & 0 \\ -\Delta & 0 & 0 & 0 \end{pmatrix}$$

Gap closes at nodes away from k=0, gap reopens at non-zero value of M/2B.
In the inverted regime, the helical edge state crossing is still robust.



Tight-binding model by X Dai, Z Fang, ...

Quantum control of the electron spin

• The electron spin can be rotated by a pure AB flux, without any interaction with the electromagnetic field.