Comment on "Chiral Suppression of Scalar-Glueball Decay"

In a recent Letter, based on an effective Lagrangian, Chanowitz [1] showed that in the limit that the mass m_q of a light quark q goes to zero, the decay amplitude for a scalar glueball G_s decaying into $q\bar{q}$ goes to zero, and conjectured further that this chiral suppression also occurs at the hadron level for G_s decays into $\pi\pi$, KK with the ratio of these two branching ratios to be of the order $\mathcal{O}(m_{u,d}^2/m_s^2)$ for finite quark masses. Here we show that the decay $G_s \to q\bar{q}$ is forbidden in the chiral limit in QCD without assumptions. More essentially, we show that this chiral suppression may be spoiled and may not materialize itself at the hadron level.

A glueball here is assumed to be a pure gluonic state. It decays into a $q\bar{q}$ pair through a multigluon annihilation process. The decay amplitude for $G_s \to q(p_1)\bar{q}(p_2)$ can be written as a product of a spinor pair $\bar{u}(p_1)$ and $v(p_2)$ with a product of any number of γ matrices sandwiched between the spinors. Because of vectorlike coupling in QCD, for $m_q = 0$ the number of the γ matrices is an odd number which can always be reduced to one γ matrix. Therefore the amplitude can be written as

$$\mathcal{T}_{q\bar{q}} = \bar{u}(p_1)\gamma_\mu A^\mu v(p_2).$$

Lorentz covariance of the amplitude then dictates $A^\mu(p_1, p_2)$ to be of the form $a_1 p_1^\mu + a_2 p_2^\mu$. Therefore in the chiral limit $m_q = 0$, $\mathcal{T}_{q\bar{q}} = 0$. The result also applies to a pseudoscalar glueball decays into a $q\bar{q}$ pair.

To study whether there is a chiral suppression in $G_s \to \pi\pi$, KK or not, we work with an effective Lagrangian, $L_s = f_g G^{a,\mu\nu} G^a_{\mu\nu} G_s$, as in [1], and employ QCD factorization [2] to calculate the amplitude $\mathcal{T}_{\pi\pi}$ for $G_s \to \pi^+\pi^-$. To the leading twist-2 order, there are two diagrams with the two gluons splitting into two quarks and two antiquarks, and then form two pions. The two gluons are off-shell by the scale at order of M_{G_s}. A direct calculation gives

$$\mathcal{T}_{\pi\pi} = -\alpha_s f_g \frac{8\pi}{9} f_\pi^2 \int_0^1 du_1 du_2 \phi_{\pi^+}(u_1)\phi_{\pi^-}(u_2) \times \left(\frac{1}{u_1 u_2} + \frac{1}{(1-u_1)(1-u_2)}\right)[1 + \mathcal{O}(\alpha_s, \lambda/M_{G_s})],$$

where ϕ_π is normalized as $\int du \phi_\pi(u) = 1$. $u_i (i = 1, 2)$ is the momentum fraction carried by the antiquark in the meson. In the above, λ can be any soft scale, such as quark mass, $\Lambda_{\rm QCD}$ and m_π. Clearly, $\mathcal{T}_{\pi\pi}$ is not zero in the chiral limit $m_q = 0$.

The amplitude for $G_s \to K^+K^-$ decay can be obtained by replacing quantities related to π by those related to K correspondingly. We would obtain $R = B(G_s \to \pi\pi)/B(G_s \to KK) \approx f_\pi^4/f_K^4 = 0.48$, which is substantially different from 1. This suppression is much milder compared with the one at the quark level. This is due to the fact that in perturbative QCD (pQCD) calculation the decay of $G_s \to \pi\pi$, KK is related to the coupling of G_s to two pairs of $q\bar{q}$ compared with conjectured by Chanowitz in [1], where it is assumed that G_s just couples to one $q\bar{q}$ pair. We should point out that whether the chiral suppression at quark level can be realized still waits for better nonperturbative calculation for the direct two quark hadronization into $\pi\pi$ and KK. If the pQCD contribution dominates, the result of $R \approx f_\pi^4/f_K^4$ can be obtained without the assumption of the effective Lagrangian. Because glueball is a pure gluon state, the amplitude of the decay $G_s \to \pi^+\pi^-$ can always be written with QCD factorization as $T_{\pi\pi} = f_\pi^2 H_g \otimes \phi_{\pi^+} \otimes \phi_{\pi^-}$, where the higher-twist effects related to π's are neglected and H_g consists of some perturbative coefficient functions and some quantities related to the structure of G_s. Although H_g is unknown, one can easily find the result of $R \approx f_\pi^4/f_K^4$.

The $f_0(1710)$ is a candidate for scalar glueball. Early measurement obtained $R \leq 0.11$ [3], and a larger one by the BES Collaboration [4] $R = 0.41^{+0.11}_{-0.17}$ recently. It is interesting to notice that the later is consistent with our result and may favor that the $f_0(1710)$ is a glueball. However, one should remember that the prediction $R \approx f_\pi^4/f_K^4$ can have substantial nonperturbative corrections and there may be further complication by mixing effects of a glueball with $q\bar{q}$ states. A more detailed study can be found in [5].

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[1] M. S. Chanowitz, Phys. Rev. Lett. **95**, 172001 (2005).
[2] S. J. Brodsky and G. P. Lepage, Phys. Rev. D **24**, 2848 (1981); G. P. Lepage and S. J. Brodsky, Phys. Rev. D **22**, 2157 (1980).
[3] W.-M. Yao *et al.* (Particle Data Group), J. Phys. G **33**, 1 (2006).
[4] M. Ablikim *et al.* (BES Collaboration), Phys. Lett. B **642**, 441 (2006).
[5] K. T. Chao, Xiao-Gang He, and J. P. Ma, hep-ph/0512327.

